



**University of Saskatchewan IEEE Student Branch**

# **ELECTRICAL ENGINEERING 4<sup>th</sup> YEAR EXAM FILE**

**(Term 1)**

**2003 Edition**

**Includes:**

EE 441  
EE 444  
EE 456  
EE 481

**Prepared for you by the IEEE**

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2. The data of the sample power system shown in Figure 2 are given in Tables 1 and 2. Using Gauss-Seidel iterative algorithm, perform 2 iterations and check the convergence after each iteration. Use a voltage magnitude tolerance of 0.001, an acceleration factor of 1.6 and 100 MVA base.

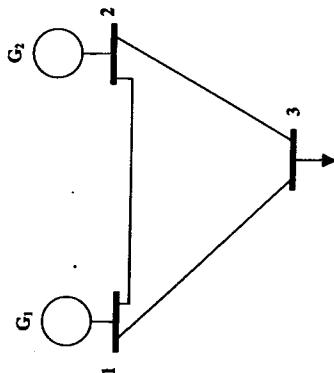


Fig. 2

Table 1: Impedances of the sample power system in p.u. on a 100 MVA base

Bus Code: p - q	Impedance $Z_{pq}$	Line charging $0.5Y_{pq}$
1-2	$0.04 + j0.16$	$j0.15$
1-3	$0.02 + j0.08$	$j0.07$
2-3	$0.05 + j0.12$	$j0.08$

Table 2: Scheduled generation and loads and magnitudes of bus voltages for the sample power system.

Bus code p	Bus voltage	Generation		Load	
		MW	MVAR	MW	MVAR
1	1.04	?	?	0	0
2	1.02	40	?	0	0
3	?	0	0	100	40

(12 Marks)

3. In the system shown in Figure 3, a three-phase fault occurred on one of the transmission lines just after the circuit breaker. Find the following:
- The critical clearing angle in degrees.
  - The critical clearing time in seconds.
  - The generator speed at the instant of clearing in radians per second.

$$x_d' = j0.4 \text{ p.u.}, \quad x_{T1} = j0.8 \text{ p.u.}, \quad x_{T2} = j0.1 \text{ p.u.}, \quad M = 7 \text{ sec}$$

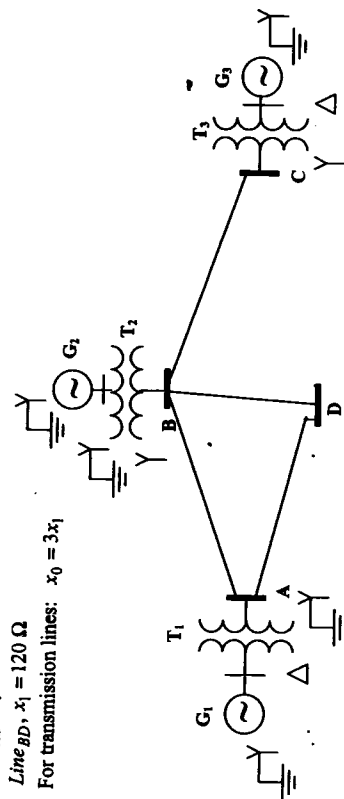
Instructor: S.O. Faried  
Duration: 3 hours

December 7, 2002

- 1- Consider the power system shown in Fig. 1. Use a power base of 500 MVA and network reduction to calculate the fault current in Amperes and the line-to-line voltages at the fault point for a sustained single line-to-ground fault at bus D.

- $G_1$ : 500 MVA, 13.8 kv,  $x_d'' = 0.2 \text{ p.u.}$ ,  $x_2 = 0.2 \text{ p.u.}$  and  $x_0 = 0.1 \text{ p.u.}$   
 $G_2$ : 600 MVA, 26 kv,  $x_d'' = 0.15 \text{ p.u.}$ ,  $x_2 = 0.15 \text{ p.u.}$  and  $x_0 = 0.1 \text{ p.u.}$   
 $G_3$ : 400 MVA, 13.8 kv,  $x_d'' = 0.2 \text{ p.u.}$ ,  $x_2 = 0.2 \text{ p.u.}$  and  $x_0 = 0.1 \text{ p.u.}$   
 $T_1$ : 500 MVA, 13.8 kv/500 kv,  $x = 0.1 \text{ p.u.}$   
 $T_2$ : 600 MVA, 26 kv/500 kv,  $x = 0.1 \text{ p.u.}$   
 $T_3$ : 500 MVA, 13.8 kv/500 kv,  $x = 0.1 \text{ p.u.}$   
Line AB,  $x_1 = 50 \Omega$   
Line BC,  $x_1 = 80 \Omega$   
Line AD,  $x_1 = 80 \Omega$   
Line BD,  $x_1 = 120 \Omega$

Mid term: Solve above using bus admittance matrix. Calculate bus voltage at bus A



For transmission lines:  $x_0 = 3x_1$   
10000 MVA short circuit capacity  
 $x_{1system} = x_{2system}$ ,  $x_{0system} = 0.5 x_{1system}$

Fig. 1

(12 Marks)

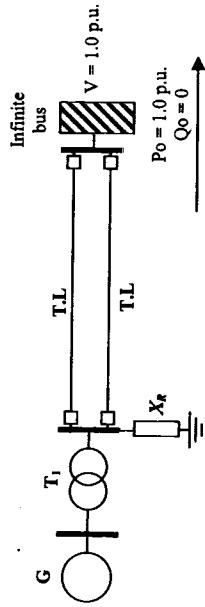


Fig. 6

(6 Marks)

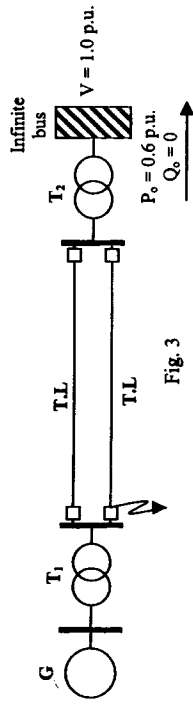


Fig. 3

(12 Marks)

4. In the system shown in Figure 4, a three-phase fault occurred on one of the transmission lines at the middle point. The switch S is opened simultaneously with circuit breakers A and B. Find the critical clearing angle.

$$x_d' = j0.4 \text{ p.u.}, \quad X_C = -j0.1 \text{ p.u.}, \quad x_{T,L} = j1.0 \text{ p.u. (each of the four sections)}$$

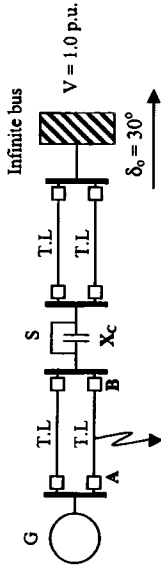


Fig. 4

(12 Marks)

5. Consider the system shown in Figure 5. Using the equal area criterion, discuss whether the transformer neutral reactance  $X_{Tn}$  improves or degrades the system transient stability.

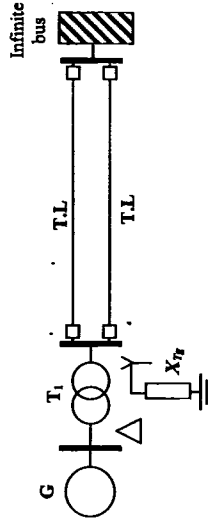


Fig. 5

(6 Marks)

6. Consider the system shown in Figure 6. Find the synchronizing power and the natural frequency of free oscillations.

$$x_d = j1.0 \text{ p.u.}, \quad x_{T,L} = j0.8 \text{ p.u.}, \quad x_{T1} = j0.1 \text{ p.u.}, \quad x_R = j0.5 \text{ p.u.}, \quad M = 7 \text{ sec}$$

October 25, 2001

1. For the system shown in Fig. 1, sketch the root locus showing all the pertinent characteristics.

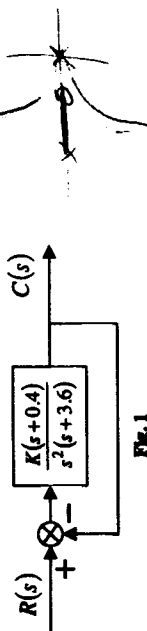


Fig. 1

2. Consider the closed-loop transfer function:

$$\frac{C(s)}{R(s)} = \frac{0.25K(s+0.435)}{s^4 + 3.456s^3 + 3.457s^2 + (0.719 + 0.25K)s + (0.0416 + 0.109K)}$$

$$K > 0.04$$

Find the range of  $K$  that ensures that the closed-loop control system is stable.

3. Consider the control system shown in Fig. 2(a). Design a rate feedback compensation, as shown in Fig. 2(b), to reduce the settling time by a factor of 4 while continuing to operate the system with the same overshoot.

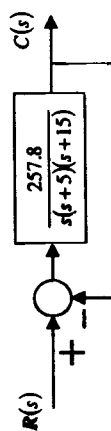


Fig. 2(a)

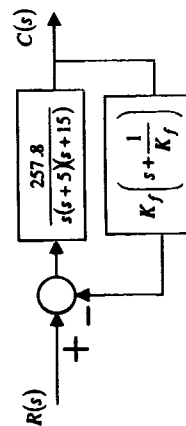


Fig. 2(b)

4. Fig. 3 shows open-loop poles and zeros. There are two possibilities for the sketch of the root locus. Sketch each of the two possibilities. Be aware that only one can be the *real* locus for specific open-loop pole and zero values.

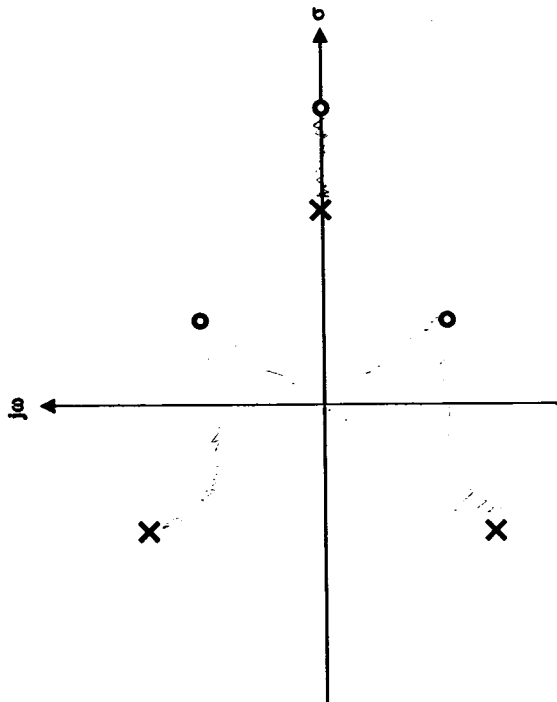
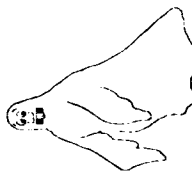


Fig. 3



Instructor: Sherif O. Faried  
A one formula sheet is allowed

Duration: 90 minutes  
October 23, 2000

- For the system of Figure 1, find the values of  $K_1$  and  $K_2$  to yield a peak time of 1 second and a settling time (2% criterion) of 2 seconds for the closed-loop system's step response.

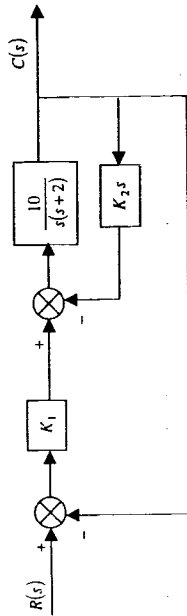


Figure 1.

- Use the Routh-Hurwitz criterion to find the range of  $K$  for which the system of Figure 2 is stable.

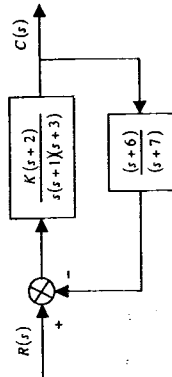


Figure 2.

- For the system shown in Figure 3, sketch the root locus showing all the pertinent characteristics and find the range of  $K$  within the system is stable.

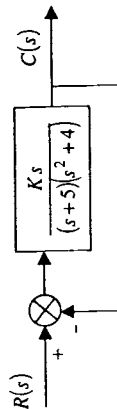
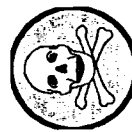


Figure 3.



Instructor: Sherif O. Faried  
A one formula sheet is allowed

Duration: 3 hours  
December 2000

- Find the following for the system shown in Figure 1:

- The transfer function  $T(s) = \frac{C(s)}{R(s)}$ .
- The damping ratio, percent overshoot, settling time (2% criterion), peak time and rise time.

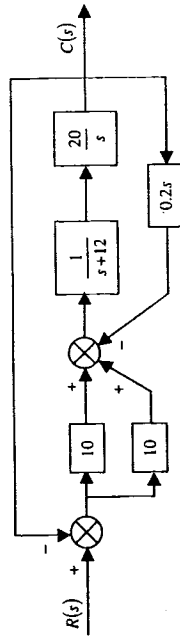


Figure 1

(10 Marks)

- Consider the control system shown in Figure 2.

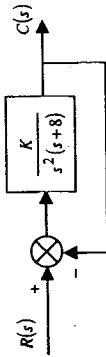


Figure 2

- Sketch the root locus and indicate *all* pertinent characteristics of the locus. Discuss the effect of the gain  $K$  on the system stability.
- If  $K = 4$ , design a compensator such that the dominant closed loop poles are located at  $s = -1 \pm j\sqrt{3}$ . Your design should lead to the maximum possible value of the static velocity error constant. Determine this maximum value.
- Sketch the root locus of the new compensated system and indicate *all* pertinent characteristics of the locus.

(16 Marks)

- Consider a unity negative feedback system with

$$G(s) = \frac{K}{(s+3)(s+5)}$$

Show that the system cannot operate with a settling time (2% criterion) of 0.667 second and a percent overshoot of 1.5% with a simple gain adjustment.

(8 Marks)

4. For the system shown in Figure 3:

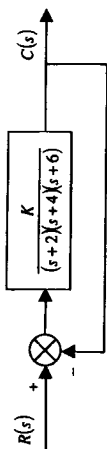


Figure 3

- Sketch the Bode plots of the open-loop transfer function.
- Sketch the Nyquist diagram.
- With the help of the Nyquist diagram, find analytically the range of gain  $K$ , for stability. (a zero mark will be given if you use Routh's stability criterion).
- Find the gain margin if  $K = 100$ .

(10 Marks)

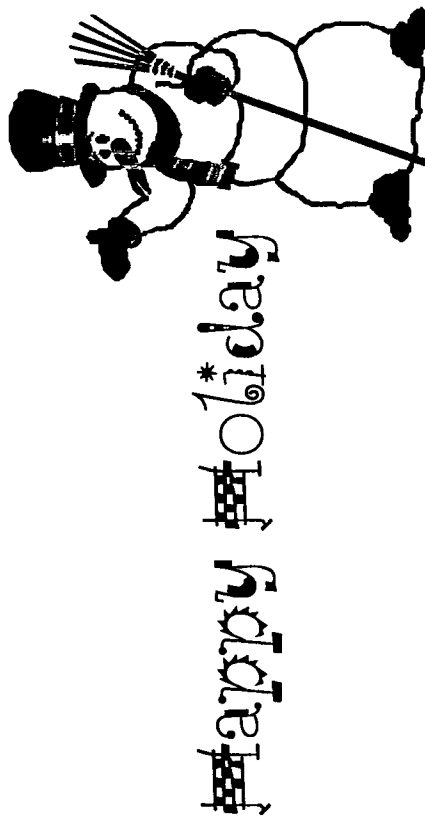
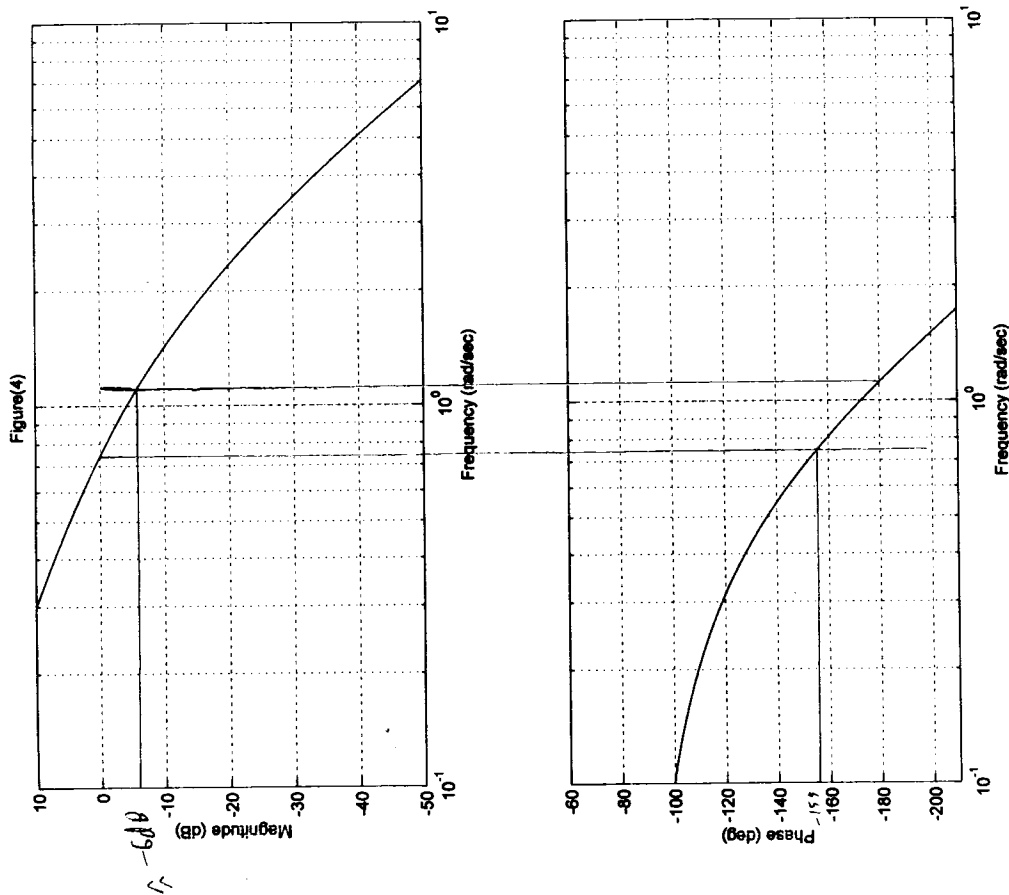
5. Consider a system having the open-loop transfer function

$$GH(s) = \frac{1}{s^4(s+p)}, \quad p > 0.$$

- Sketch the Bode plots of the open-loop transfer function.
  - Sketch the Nyquist diagram.
  - Determine  $N$ ,  $P$  and  $Z$  and discuss the stability of the system.
6. The Bode plots for a plant  $G(s)$ , used in a unity negative feedback system are shown in Figure 4. Find the gain margin and the phase margin.

(8 Marks)

(8 Marks)



1. A load added to a truck results in a force  $F$  on the support spring and the tire flexes as shown in Fig. P2.47(a). The model for the tire movement is shown in Fig. P2.47(b).

a) Determine the differential equation relating the displacement of the mass  $M$  and the applied force  $F$ .

b) Determine the transfer function  $X_1(s)/F(s)$ .

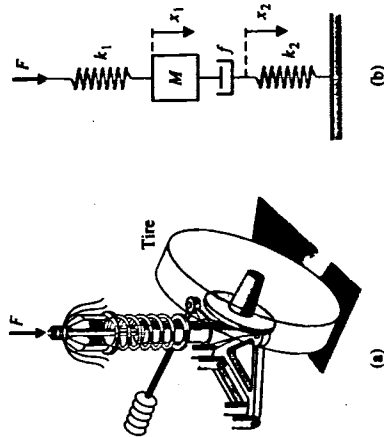


FIGURE P2.47 Truck support model.

2. An ideal set of gears is connected to a solid cylinder load as shown in Fig. P2.45. The inertia of the motor shaft and gear  $G_2$  is  $J_m$ . Determine (a) the inertia of the load  $J_L$  and (b) the torque  $T$  at the motor shaft. Assume the friction at the load is  $f_L$  and the friction at the motor shaft is  $f_m$ . Also assume the density of the load disk is  $\rho$ .

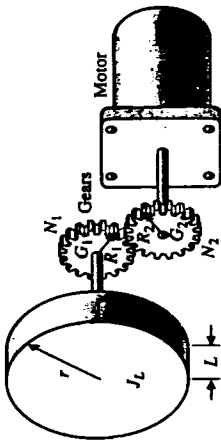


FIGURE P2.45 Motor, gears, and load.

# EE 410.3 Controls I

Quiz # 2 October 1997

Do Both Questions:

1. A control system has the structure shown in Fig. 1.
  - a) Determine the closed loop transfer function  $C(s)/R(s)$  using the method of block diagram manipulation.
  - b) Select gains  $K_1$  and  $K_2$  so that the closed loop response to a step input is critically damped with two equal roots at  $s = -10$ .

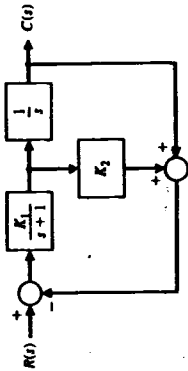


Figure 1.

2. The circuit shown in Fig. 2 is called a lead-lag filter.

- a) Find the transfer function  $V_2(s)/V_1(s)$  using the signal flow graph method and Mason's rules. (Draw the flow graph and indicate how you find the transfer function.)

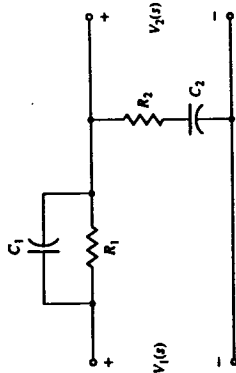
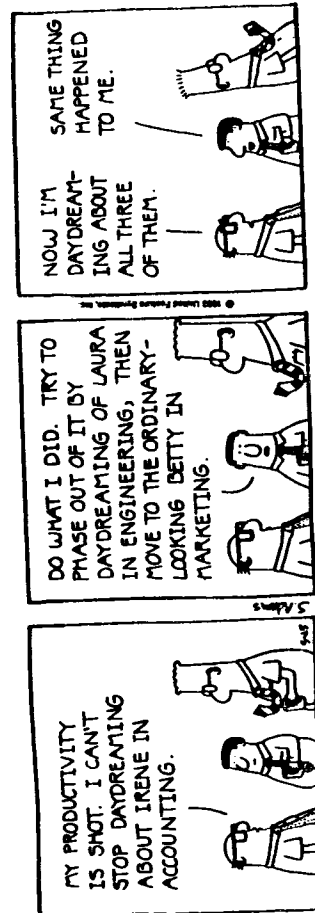


Figure 2.

- b) Confirm the result of part a) using any other method to find the same transfer function.



Notes:  
Instructor: H. Wood  
Time: 85 minutes  
Notes: 2 or 3 pages  
Marks: As indicated; Do all 3 questions.

November 1996

1. The objective of this question is to design a controller for the system illustrated in Figure 1. The controller is required to have a DC gain of  $K$ , and must have a single pole at a location  $b$  on the left hand side of the  $s$ -plane. To solve for the two unknown factors in the controller, two design criteria are given. The steady state error in response to a unit step input is 20%, and the system must be stable.

- What is the expression for the transfer function of the controller itself?
- Show that the DC gain of your controller is in fact  $K$ .
- Find the closed loop transfer function  $T(s)$ .
- What is the expression for the steady state error of the closed loop system in response to a step input?
- Use the steady state error limit of 0.2 to evaluate one of the controller unknowns (it should be clear from the expression for the s.s.e. which one).
- Use the stability criterion to find the range of acceptable values for the second controller unknown.

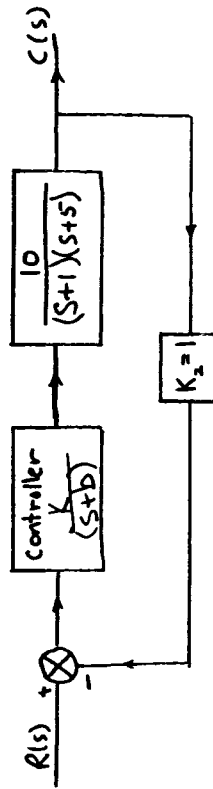


Figure 1

2. A controller with a single pole at  $s = -100$  and a gain factor of  $K$  is used to provide an input signal to a plant. Unity gain negative feedback is established by comparing the output signal  $C(s)$  with the reference input signal  $R(s)$ . When a step input is applied to the OPEN loop system, the response is as shown in Figure 2.

- Assume the response is approximately second order. What are the natural frequency and the damping ratio for the plant?
- What is the value of the gain factor for the controller?  
(Hint: Use the Final Value Theorem and the illustrated response)
- Show all of the root locations in the  $s$ -plane for the open loop system.
- Is the assumption made in part a) justified? Why or why not?
- Now connect the feedback and determine the closed loop transfer function.
- Again assuming the system is approximately second order, what is the natural frequency of the closed loop system? How does it compare with the open loop system frequency? Do you expect this result for the comparison? Why?

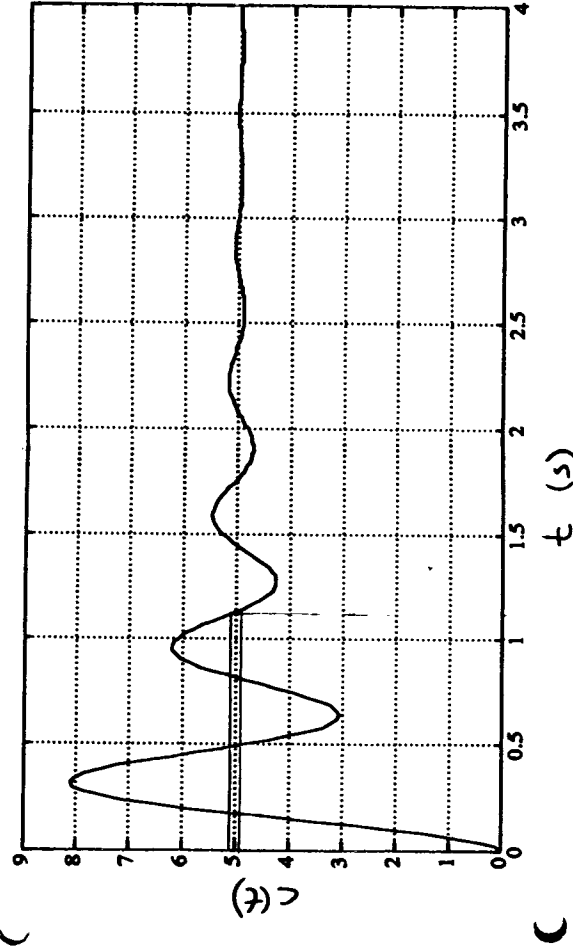


Figure 2.



3. The operational amplifier circuit in Figure 3 consists of a differential amplifier followed by two separate but equivalent filter units. The differential amplifier, in the configuration shown, multiplies the voltage difference at its input terminals A and B by the DC gain factor  $K$ . The operational amplifier has an effective time constant of 0.01 seconds. The filter resistor values are each 10 kOhms and the capacitor values are 2.0 microfarads.

- Draw the block diagram of the closed loop system.
- What is the maximum value of the DC gain of the amplifier for stability?
- At the maximum gain, at what frequency will the circuit oscillate?

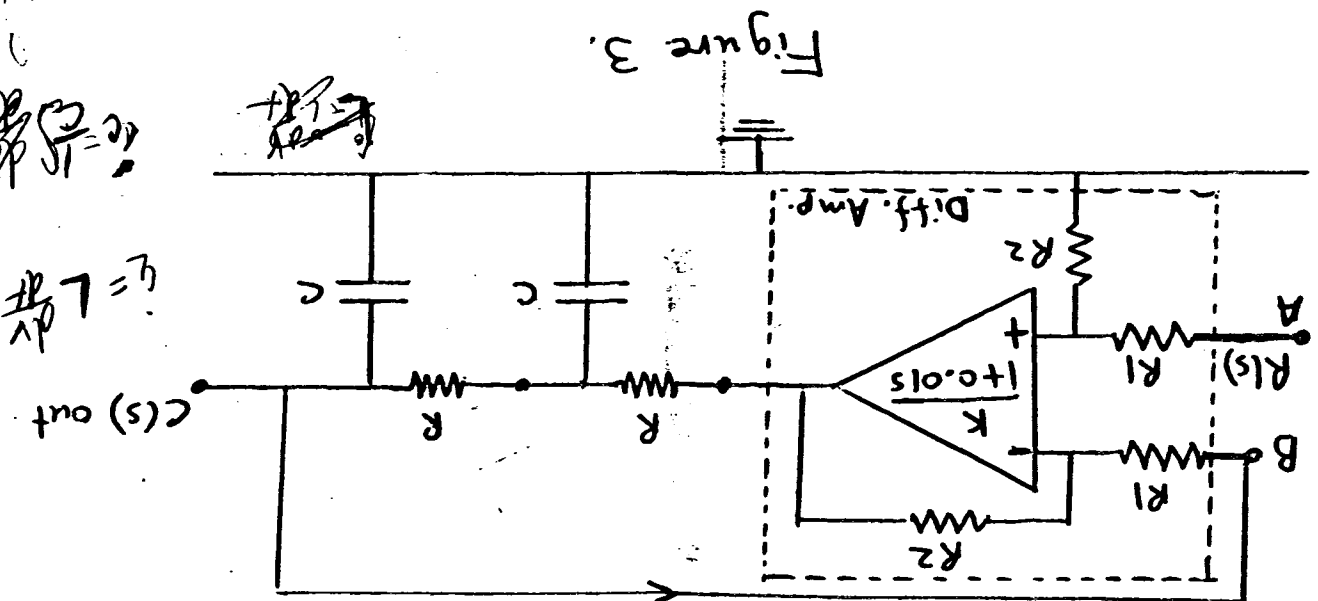


Figure 3.

$$V = \left( R + \frac{1}{sC} \right) \cdot \frac{V \cdot \frac{s}{R + \frac{1}{sC}}}{\frac{sRC + 1}{V}}$$

$$K \cdot \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \cdot \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

$$\frac{1}{sC}$$

$$L = \frac{dV}{dt}$$

$$C = \frac{1}{sC}$$

October 27, 1999

Instructor: S.O. Faried

Time: 90 minutes

Note: One formula sheet is allowed

1. Consider the system shown in Figure 1. Determine the range of values of  $K$  for which the system is stable.

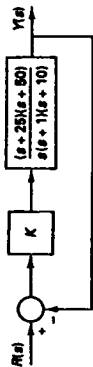


Figure 1

2. Sketch the root locus for a unity-feedback control system whose forward transfer function is given by

$$G(s) = \frac{K}{(s+2)^2}$$

At what value of  $K$  does the system become unstable, and where does the root locus intersect the  $j\omega$  axis when this occurs?

3. Sketch the root locus for a unity-feedback control system whose forward transfer function is given by

$$G(s) = \frac{K(s+2)}{s^2(s+18)}$$

- (i) Determine the location of the roots when all three roots are all real and equal.
- (ii) Find the gain when all the roots are real and equal.

The End

EE 410 Midterm

Your task today is to design a control system for a new electric car. The car, with a total mass of 800 kg, is battery operated and all of the controls are electrical or electronic. The car is driven by an electric motor whose output torque is proportional to the current through the motor. The motor is connected to the wheels through a gear reduction of 5:1 (motor shaft turns 5 times as fast as the axle), and the wheel diameter is 36 cm. The electric motor can be modelled as a resistance  $R$  in series with an inductance  $L$ . The vehicle experiences air friction and rolling friction, all combined in one term that is directly proportional to speed.

To control the speed of the vehicle, a power control unit outputs a voltage to the motor that is directly proportional to the angular position of a manually operated dial on the control unit.

**Note:** This question has many parts; each part is really a continuation of the same problem, but, it is not necessary to get each part correct to proceed to the next part. Each succeeding part starts from an assumed solution to the previous part that is given to you. This solution is not necessarily the actual solution to the previous part, but gives everyone the same starting point for the next part. Even if you think you have the correct solution to a part, do not use your solution for the next part, but instead use the one given to you.

1. Assume the vehicle is at rest at time  $t=0$  and the dial is set to 0.

Draw a sketch of the system to help you visualize what is going on.

- a) Develop the differential equation that relates the torque produced by the motor to the position of the vehicle. (ignore rotational inertias)
- b) Develop the differential equation that relates the angular position of the 'speed dial' on the controller to the motor torque.
- c) Put these equations together to give an equation relating the dial setting to the vehicle position.

2. Assume that the solution to 1 c) is as follows: ( $d$  is dial position,  $x$  is vehicle position).

$$d(t) = A x'''(t) + B x''(t) + C x'(t) \quad \text{where } x' \text{ represents } dx/dt.$$

- a) Determine the Laplace transform expression relating the variables  $x$  and  $d$ , assuming zero initial state for the system.
- b) Determine the Laplace transform expression relating the vehicle speed  $v$  to the dial setting, now assuming that the vehicle is moving at uniform speed  $v(0)$  at time  $t=0$ .

**Note:** Use degrees throughout; do not change to radians.

University of Saskatchewan  
College of Engineering  
EE 444.3: Electrical Machines II  
Midterm Examination

Instructor: Dr. N. Kar  
Time: 1 hour & 20 min.  
Note: One sheet of handwritten formulas permitted

October 29, 2002

Marks

- 20 1. (a) Calculate the force produced on the moving part of the shown unipivot relay mechanism (Fig. 1) where the motion may be assumed to be linear. The coil has 1000 turns and the DC current flowing in it is 1.0 A. Neglect fringing and leakage flux, and assume that all the energy is stored in the air-gap.

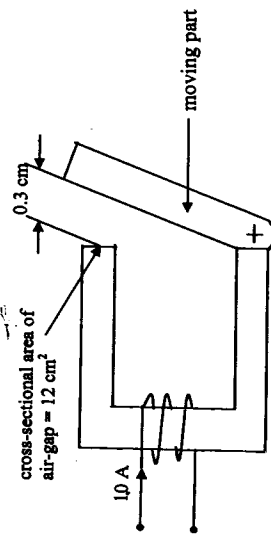


Fig. 1. Relay mechanism.

- (b) If the following factors:

- (i) the leakage flux
- (ii) the fringing effect
- (iii) the iron path of the magnetic path

are not neglected, describe using literature the effect of these factors on the value of the force calculated in (a).

- (c) Answer whether the following statements are true or false.

- (i) If the magnetization curve of an electromagnetic device is nonlinear, the energy stored in the magnetic field is smaller than the coenergy.
- (ii) The synchronous reactance of a synchronous generator is larger than its leakage reactance.
- (iii) A synchronous generator operating at lagging PF (power factor) is underexcited.

- 30 2. A 480 V, 60 Hz,  $\Delta$ -connected, 4-pole synchronous generator has the open-circuit characteristic shown in Fig. 2. This generator has a synchronous reactance of  $0.11 \Omega$  and an armature resistance of  $0.016 \Omega$ . At full-load, the machine supplies 1200 A at 0.8 PF lagging. Under full-load conditions, the friction and windage losses are 40 kW, and the core losses are 30 kW. Ignore any field circuit losses.

- (a) What is the speed of rotation of this generator?
- (b) How much field current must be supplied to the generator to make the terminal voltage 480 V at no load?
- (c) If the generator is now connected to a load and the load draws 1200 A at 0.8 PF lagging, how much field current will be required to keep the terminal voltage equal to 480 V. Draw the phasor diagram.
- (d) How much power is the generator now supplying? How much power is supplied to the generator by the prime-mover? What is the machine's overall efficiency?
- (e) If the generator's load were suddenly disconnected from the line, what would happen to its terminal voltage?
- (f) Finally, suppose the generator is connected to a load drawing 1200 A at 0.8 PF leading. Draw the phasor diagram. How much field current would be required to keep the terminal voltage at 480 V?

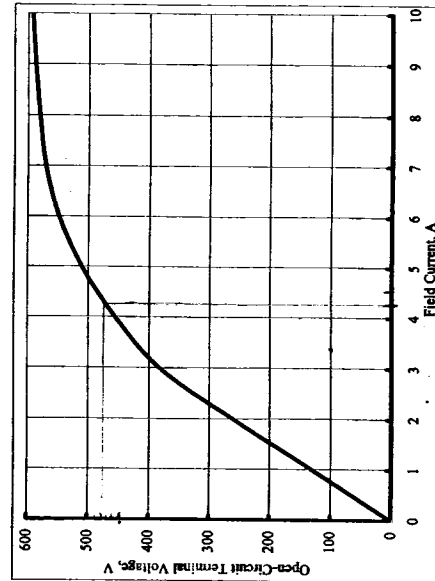


Fig. 2. Open-circuit characteristic of the generator in Question 2.

— The End —

- 20 3. (a) What are the advantages and disadvantages of brushless dc motors compared to ordinary brush dc motors?
- (b) A 460-V, 25-hp, 60-Hz, 4-pole, Y-connected, wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641 \, \Omega \quad R_2 = 0.332 \, \Omega$$

$$X_1 = 1.106 \, \Omega \quad X_2 = 0.464 \, \Omega \quad X_M = 26.3 \, \Omega$$

- i) What is the maximum torque of this motor? At what speed and slip does it occur?
- ii) What is the starting torque of this motor?
- iii) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

- 15 4. (a) Neglecting the stator resistance, show that the active power output of a cylindrical-rotor synchronous generator connected to an infinite bus is given by

$$P = \frac{E_f V_t}{X_s} \sin \delta$$

- (b) Describe the effect of the excitation on the synchronous generator performance using phasor diagram when the generator real power output, frequency and terminal voltage are constant.

- 25 5. A 2000-hp, 1.0-power factor, 3-phase, Y-connected, 2300-V, 30-pole, 60-Hz synchronous motor has a synchronous reactance of 1.95  $\Omega$ /phase. For this problem all losses may be neglected.
- (a) Compute the maximum torque which this motor can deliver if it is supplied with power from a constant frequency source, commonly called an *infinite bus*, and if its field excitation is constant at the value which would result in 1.0 power factor at rated load.

- (b) Instead of the infinite bus of part (a) suppose that the motor is supplied with power from a 3-phase, Y-connected, 2300-V, 1750-kVA, 2-pole, 3600-r/min turbine generator whose synchronous reactance is 2.65  $\Omega$ /phase. The generator is driven at rated speed, and the field excitations of the generator and motor are adjusted so that the motor runs at 1.0 power factor and rated terminal voltage at full load. The field excitations of both machines are then held constant, and the mechanical load on the synchronous motor is gradually increased. Compute the maximum motor torque under these conditions and the terminal voltage when the motor is delivering its maximum torque.

— THE END —

Instructor: Dr. N. Kar

Time: 3 hours

Note: Two sheets of handwritten formulas permitted.

December 20, 2002

Marks

- 20 1. The dimensions of electromagnet shown in Fig. 1 are in centimetre (cm) and the depth of the core and the armature is 5 cm. The coil has 1000 turns. Assuming that the permeability of the magnetic material is very large relative to air ( $\mu_r = 4\pi \times 10^{-7}$  H/m) and neglecting the leakage flux and the fringing of flux at the air-gaps:

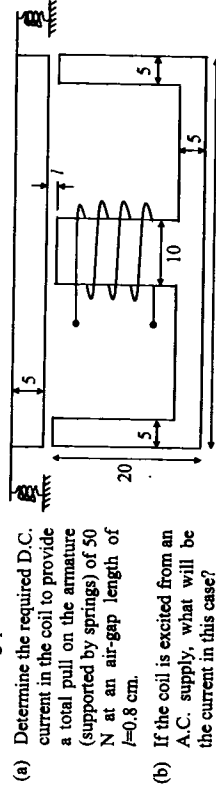


Fig. 1

- (a) Determine the required D.C. current in the coil to provide a total pull on the armature (supported by springs) of 50 N at an air-gap length of  $l = 0.8$  cm.
- (b) If the coil is excited from an A.C. supply, what will be the current in this case?
- 20 2. Fig. 2 depicts a simple, single-phase, 4-pole reluctance motor. A current of 1 A at 60 Hz is passed through its stator winding. Assuming a sinusoidal variation of inductance of this winding in terms of  $\theta$ , between the maximum value of 0.4 H and a minimum value of 0.1 H:
- (a) Derive an expression as a function of time for the torque produced by this motor.
- (b) Determine the value of the speed at which this motor will develop an average torque. What will be the maximum value of this average torque at this speed?
- (c) What are the frequencies of the time varying components of the produced torque? What are the amplitudes of these components?

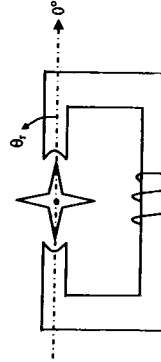


Fig. 2

Instructor: S.O. Faried  
Duration: 80 minutes

October 30, 2001

1. A 0.25 hp, 110-V, 60-Hz, four-pole capacitor-start single-phase induction motor has the following parameters and losses:

$$R_1 = 2 \Omega \quad X_{11} = 2.8 \Omega \quad X'_{12} = 2 \Omega \quad R'_2 = 4 \Omega \quad X_m = 70 \Omega$$

Core loss at 110 V = 25 W; Friction and windage = 12 W

For a slip of 0.05, compute the input current, power factor, power output, speed, torque and efficiency when the motor is running at rated voltage and rated frequency with its starting winding open.  $I = 3.57 \angle -49.8^\circ$   $P_{in} = 110 \text{ W}$   $P_{out} = 101 \text{ W}$   $\eta = 91.4\%$

2. A 3-phase, squirrel-cage induction motor has a starting torque of 1.75 p.u. and a maximum torque of 2.5 p.u. when operated from rated voltage and frequency. The full-load torque is considered as 1 p.u. of torque. Neglect stator resistance.

(a) Determine the slip at maximum torque. 0.7

(b) Determine the slip at full-load torque. 0.38

- (c) Determine the rotor current at starting in p.u. Consider the full-load rotor current as 1 p.u.  $I_r = 3.57 \angle -49.8^\circ$

3. A 500 hp, 3-phase, 2200-V, 60-Hz, 12-pole, Y-connected, wound rotor induction motor has the following parameters:

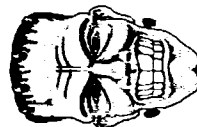
$$R_1 = 0.225 \Omega \quad R'_2 = 0.235 \Omega \quad X_{11} + X'_{12} = 1.43 \Omega \quad X_m = 31.8 \Omega$$

Use an appropriate equivalent circuit to calculate the following:

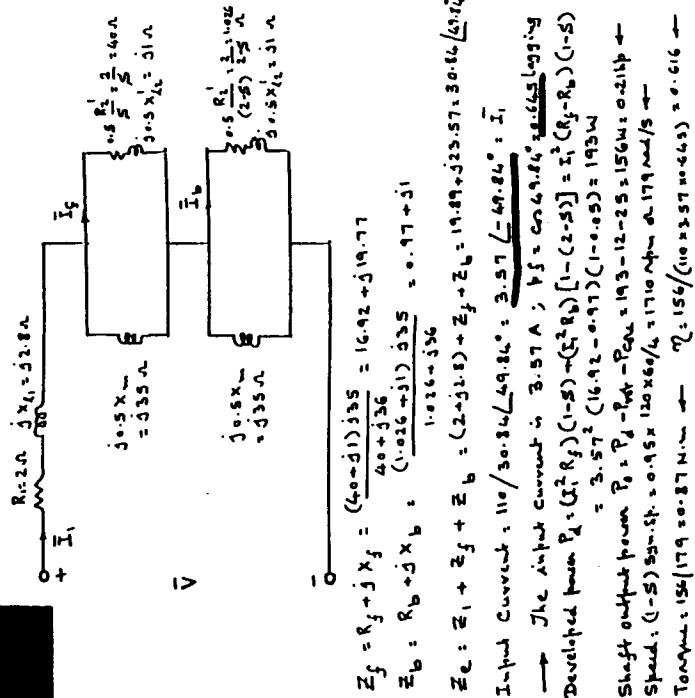
- (a) Slip at maximum torque. 0.084

(b) Maximum torque. 1054.9 N.m

- (c) Resistance that must be added to the rotor windings (per phase) to achieve maximum torque at starting.  $R_{add} = 2.81 \Omega$



S.O. Faried's Answers  
EE453 Oct 30, 2001



[2]

Neglect  $R_1$ 

$$T_{\text{starting}} = \frac{3 V_{th}^2 R_1'}{w_s [(R_1')^2 + (X_{eq})^2]} \quad \text{Eq. ①}$$

$$T_{\text{max}} = \frac{3 V_{th}^2}{2 w_s [X_{eq}]} \quad \text{Eq. ②}$$

$$S_{\text{max}} = \frac{R_1'}{X_{eq}} \Rightarrow R_1' = S_{\text{max}} X_{eq}$$

$$\frac{T_{\text{max}}}{T_{\text{starting}}} = \frac{\cancel{3 V_{th}^2}}{2 w_s X_{eq}} \cdot \frac{w_s [R_1'^2 + X_{eq}^2]}{\cancel{3 V_{th}^2} R_1'}$$

$$\frac{T_{\text{max}}}{T_{\text{st}}} = \frac{1}{2 X_{eq}} \frac{[R_1'^2 + X_{eq}^2]}{R_1'}$$

$$= \frac{1}{2 X_{eq}} \frac{[S_{\text{max}}^2 X_{eq}^2 + X_{eq}^2]}{S_{\text{max}} X_{eq}}$$

$$\frac{T_{\text{max}}}{T_{\text{st}}} = \frac{\cancel{X_{eq}^2}}{\cancel{X_{eq}^2}} \frac{[1 + S_{\text{max}}^2]}{2 S_{\text{max}}}$$

$$\frac{2.5}{1.75} = \frac{1 + S_{\text{max}}^2}{2 S_{\text{max}}}$$

$$1.75 S_{\text{max}}^2 - 5 S_{\text{max}} + 1.75 = 0$$

Solve for  $S_{\text{max}}$ 

$$S_{\text{max}} = 2.45 \text{ or } 0.408$$

$$S_{\text{max}} = 0.408$$

$$T_{\text{f.l}} = \frac{3 V_{th}^2 R_1'}{w_s \left[ \frac{R_1'^2}{S_{\text{f.l}}^2} + X_{eq}^2 \right]}$$

$$\frac{T_{\text{max}}}{T_{\text{f.l}}} = \frac{\cancel{3 V_{th}^2}}{2 w_s X_{eq}} \cdot \frac{w_s \left[ \frac{R_1'^2}{S_{\text{f.l}}^2} + X_{eq}^2 \right] S_{\text{f.l}}}{\cancel{3 V_{th}^2} R_1'}$$

$$= \frac{1}{2 X_{eq}} \frac{\left[ \frac{R_1'^2}{S_{\text{f.l}}^2} + X_{eq}^2 \right] S_{\text{f.l}}}{R_1'}$$

$$= \frac{1}{2 X_{eq}} \frac{\left[ \frac{S_{\text{max}}^2 X_{eq}^2}{S_{\text{f.l}}^2} + X_{eq}^2 \right] S_{\text{f.l}}}{S_{\text{max}} X_{eq}}$$

$$\frac{T_{\text{max}}}{T_{\text{f.l}}} = \frac{S_{\text{max}}^2 + S_{\text{f.l}}^2}{2 S_{\text{max}} S_{\text{f.l}}}$$

$$2.5 = \frac{(0.408)^2 + S_{\text{f.l}}^2}{2 * 0.408 * S_{\text{f.l}}}$$

$$S_{fL} = 1.955 \text{ or } 0.085$$

$$S_{fL} = 0.085$$

$$T \propto \frac{I_2^2 R_2'}{s}$$

$$\frac{T_{st}}{T_{fL}} = \frac{I_{2st}^2}{I_{2fL}^2} \cdot \frac{S_{fL}}{S_{st}} = \left( \frac{I_{2st}}{I_{2fL}} \right)^2 \cdot \frac{0.085}{1} = 1.75$$

$$\frac{I_{2st}}{I_{2fL}} = \sqrt{\frac{1.75}{0.085}} = 4.53, \quad I_{2fL} = 1 \text{ p.u.}$$

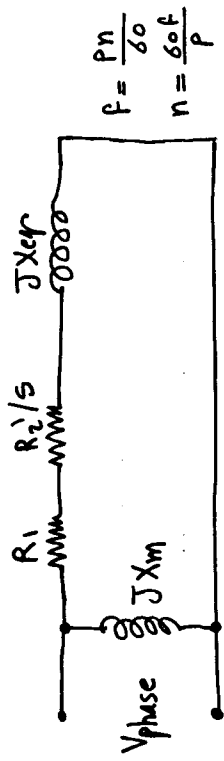
$$I_{2st} = 4.53 \text{ p.u.}$$

$$R_1 = 0.225 \Omega$$

$$R_2' = 0.235 \Omega$$

$$X_{ef} = 1.43 \Omega$$

$$X_m = 31.8 \Omega$$



$$S_{max} = \frac{R_2'}{\sqrt{(R_1)^2 + (X_{ef})^2}}$$

$$S_{max} = \frac{0.235}{\sqrt{(0.225)^2 + (1.43)^2}} = 0.1623$$

$$T_{max} = \frac{3 V_{ph}^2}{2 \omega_s [R_1 + \sqrt{(R_1)^2 + (X_{ef})^2}]}$$

$$n_s = 600 \text{ r.p.m.}, \quad \omega_s = \frac{2\pi n_s}{60} = 62.8319 \text{ rad/sec}$$

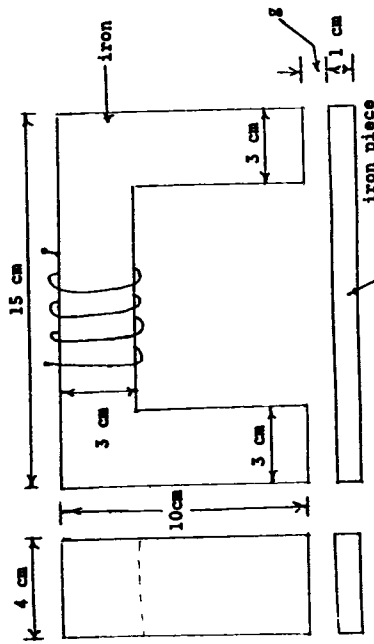
$$T_{max} = \frac{3 * (1270.1706)^2}{125.6637 [0.225 + \sqrt{(0.225)^2 + (1.43)^2}]}$$

Instructor: Dr. A. M. El-Serafi  
Note: One sheet of handwritten  
notes and formulas permitted.

November 1994

Marks

- 25 1. The exciting coil of the shown electromagnet has 1,000 turns and carries a constant current of 5A. Neglecting the leakage, fringing in the air gaps and the reluctance of the magnetic material, calculate:
- The magnetic force acting on the iron piece when the gap length  $g = 1$  cm.
  - The energy supplied by the electrical source if the iron piece is allowed to move from the above position until the air gap length becomes 0.5 cm. Neglect the resistance of the coil.
  - The mechanical work done by the iron piece for case (b).



- 5 2. How will the magnitude of the magnetic force calculated in (a) of problem (1) be changed:
- If the reluctance of the magnetic material is to be considered?
  - If the fringing flux at the air gaps is not neglected?
  - If the leakage flux is not neglected?

...(2)

November 1994

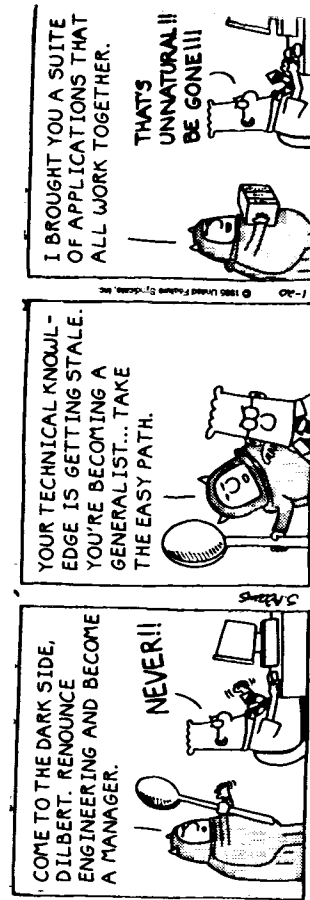
- 20 3. A 230-V, 10-hp, 60-Hz, 4-pole, star-connected, 3-phase induction motor has the following per-phase equivalent circuit parameters:

$$\begin{aligned} r_1 &= 0.36\Omega & r_2' &= 0.19\Omega & x_m &= 15.5\Omega \\ x_1 &= 0.47\Omega & x_2' &= 0.47\Omega \end{aligned}$$

Neglecting the core and mechanical losses, calculate:

- The maximum torque of this motor and the speed at which this torque occurs.
- The starting torque of this motor.

\*\*\* The End \*\*\*





Duration: 90 minutes  
October 24, 2000

Instructor: Sherif O. Faried  
A one formula sheet is allowed

1. Draw the circle diagram of a 10 hp (7.46 kW), 200 V, 60 Hz, 4-pole, Y-connected, 3-phase slip-ring induction motor with a winding ratio of unity, a stator resistance of  $0.38 \Omega$ /phase and a rotor resistance of  $0.24 \Omega$ /phase. The following are the test readings:  
No-load test: 200 V, 7.7 A,  $\cos \phi_0 = 0.195$   
Locked rotor test: 100 V, 47.6 A,  $\cos \phi_L = 0.454$   
Find:  
(a) The starting torque.  
(b) The maximum torque.  
(c) The slip for maximum torque.  
(d) The maximum power factor.

2. A 20 hp, 400 V, 60-Hz, 4-pole, Y-connected, 3-phase squirrel-cage induction motor takes 6 times the full-load current at standstill and rated voltage and develops 1.8 times full-load running torque. The full load current is 30 A.  
(a) What voltage must be applied to produce full-load torque at starting?  
(b) What will be the starting current with this new applied voltage?  
(c) Consider now that this reduced voltage is obtained using an autotransformer. What will be the supply current?

3. A 3-phase, 460 V, 1740 r.p.m. 60-Hz, 4-pole wound-rotor induction motor has the following parameters per phase:

$$R_1 = 0.25 \Omega, \quad R_2 = 0.2 \Omega, \quad X_1 = X_2 = 0.5 \Omega, \quad X_m = 30 \Omega$$

The rotational losses are 1700 watts. With the rotor terminals short-circuited, find:

- (i) Starting Torque
- (ii) Air-gap power
- (iii) Induced torque
- (iv) Slip at which maximum torque is developed
- (v) How much external resistance per phase (referred to the stator) should be connected in the rotor circuit so that maximum torque occurs at start?



Instructor: Sherif O. Faried  
Three formula sheets are allowed  
A graph paper is provided

Duration: 3 hours  
December 8, 2001

1. A 200-V, 60 Hz, six-pole, Y-connected, 10-hp (7.46 kW) slip-ring induction motor tested in the laboratory, with the following results:

No load	200 V	7.7 A	520 W
Locked rotor	100	47.6	3743

The effective stator to rotor winding ratio is 1, the stator resistance is 0.38 ohm/phase and the rotor resistance is 0.24 ohm/phase. Draw the motor circle diagram and find:

- Starting torque
- Maximum torque
- Slip for maximum torque
- Maximum power factor
- Maximum output

2. A 10-hp, four-pole, 60-Hz, three-phase induction motor develops its full-load induced torque at 3 per cent slip when operating at 60-Hz and rated voltage. The per-phase circuit model impedances of the motor are:

$R_1 = 0.36 \Omega$	$R_2 = 0.15 \Omega$	$X_m = 15.5 \Omega$
$X_1 = 0.47 \Omega$	$X_2' = 0.47 \Omega$	

Mechanical, core and stray losses may be neglected in this problem. What is the maximum torque of this motor?

3. A 208-V, four-pole, 60-Hz, Y-connected wound rotor induction motor is rated at 15 hp. Its equivalent circuit components referred to the stator winding are:

$R_1 = 0.21 \Omega$	$R_2' = 0.137 \Omega$	$X_m = 13.2 \Omega$
$X_1 = 0.442 \Omega$	$X_2' = 0.442 \Omega$	

$P_{core} = 200 \text{ W}$ ,  $P_{F\&W} = 300 \text{ W}$ . The ratio of stator to rotor turns per phase is 3.5/1.

Due to the requirements of a large starting capability, it is necessary to cause this motor to develop maximum torque at starting. How much external resistance must be added to each rotor phase to meet this requirement?

4. A salient-pole synchronous generator is connected to an infinite bus through an external reactance  $x_e = 0.2 \text{ p.u.}$  (Fig. 1). The synchronous reactances are  $x_d = 1.4 \text{ p.u.}$  and

$x_q = 0.8 \text{ p.u.}$  The generator is supplying the following active and reactive powers to the infinite bus system:  $P_o = 0.9 \text{ p.u.}$ ,  $Q_o = 0$ . The infinite bus voltage is  $V = 1.1 \text{ p.u.}$

Draw the vector diagram and calculate for this operating condition:

- The per-unit terminal and excitation voltages.
- The power angle in degrees.
- The voltage regulation.
- The reluctance power in per-unit.
- The per-unit maximum power the generator can deliver without losing synchronism.

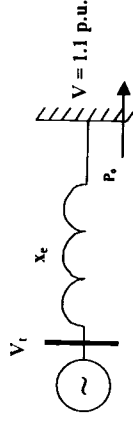


Fig. 1

- A three-phase, Y-connected, round-rotor synchronous motor has a synchronous reactance of 1.0 p.u. and an armature resistance of 0.05 p.u./phase. Do not neglect the armature resistance in your calculations.
  - If the motor takes a line current of 1.0 p.u. at 0.8 p.f. lagging from an infinite bus of 1.0 p.u. voltage, calculate the excitation voltage and the power angle.
  - If the motor is operating on load with a power angle of  $-21.1233$  degrees and the excitation is so adjusted that the excitation voltage is equal to 1.6481 p.u., determine the armature current and the power factor of the motor.
- A 13.8 kV, 10 MVA, 60-Hz, 2-pole, Y-connected turbine-generator has a synchronous reactance of 22.8528 ohm/phase and a negligible armature resistance. This generator is operating in parallel with a very large power system with a voltage magnitude of 13.8 kV.
  - What is the magnitude of the excitation voltage (in p.u.) at rated current and 0.8 p.f. lagging.
  - What is the power angle of the generator under the conditions of (a)
  - If the field current is constant, what is the maximum power (in p.u.) possible out of this generator?
  - At the absolute maximum power possible, how much reactive power (in p.u.) will this generator be supplying or consuming? Sketch the corresponding phasor diagram.
- A three-phase synchronous generator is operating at a lagging power factor condition on an infinite bus. Treat the machine as lossless. If the prime mover power supplied to the generator is increased, but the field current is adjusted so that the output reactive power is unchanged, draw the vector diagram and qualitatively describe the changes in  $I_a$ ,  $E_f$ ,  $\phi$  and  $\delta$ .

Duration: 3 hours  
December 2000

Instructor: Sherif O. Faried  
A one formula sheet is allowed  
A graph paper is provided

1. Prove that if the stator resistance of a three-phase induction motor is neglected ( $R_1 = 0$ ), the torque/slip curve of such a motor can be expressed by the relation:

$$\frac{T}{T_{\max}} = \frac{\frac{s}{s_{\max T}} + \frac{s_{\max T}}{s}}{2}$$

where  $s$  and  $s_{\max T}$  are the slips corresponding to  $T$  and  $T_{\max}$  respectively.

2. The approximate per-phase equivalent circuit for a 3-phase, 4-pole, 60-Hz, 1710 rpm double-cage rotor induction machine is shown in Fig. 1. The standstill rotor impedances referred to the stator are as follows:

Outer cage:  $4.0 + j1.5 \Omega$   
Inner cage:  $0.5 + j4.5 \Omega$

If the stator impedance is neglected,

- (a) Determine the ratio of currents in the outer and inner cages for standstill and full-load conditions.

- (b) Determine the starting torque of the motor as percent of full-load torque.

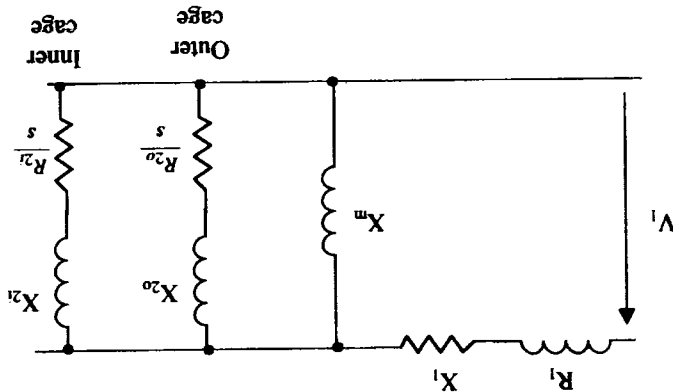


Fig. 1

3. A salient-pole synchronous generator supplies a load at a unity power factor to an infinite bus whose voltage is 1.05 p.u. The generator e.m.f ( $E_f$ ) under this condition is 1.4 p.u. If  $X_d = 0.95$  p.u. and  $X_q = 0.65$  p.u.
  - (a) Draw the vector diagram under this operating condition.
  - (b) Calculate the power delivered to the infinite bus and the load angle.

4. The following data are obtained from the open-circuit and short-circuit characteristics of a three-phase, wye-connected, four-pole, 150-MW, 0.9-p.f., 12.6-kV, 60-Hz, hydrogen-cooled turbine-generator with negligible armature resistance:

Open-circuit characteristic									
Field current, A	Line-to-line voltage, kV								
	200	300	400	500	600	700	800	900	1150
	1500	1150	900	800	700	600	500	400	300
	17.3	15.58	14.2	13.5	12.6	11.3	9.8	7.8	5.8
Short-circuit characteristic									
Field current, A	Armature current, A								
	350	4043	700	8086					

Determine:

- The unsaturated synchronous reactance in p.u.
- The saturated synchronous reactance in p.u. and the short-circuit ratio.
- The estimated field current and voltage regulation for rated voltage, rated current and a unity p.f. operation.
- The power angle under this condition.

5. A salient-pole synchronous generator is connected to an infinite bus through an external reactance  $x_e = 0.2 \text{ p.u.}$  (Fig. 2). The synchronous reactances are  $x_d = 1.4 \text{ p.u.}$  and  $x_q = 0.8 \text{ p.u.}$  The generator is supplying the following active and reactive powers to the infinite bus system:  $P_o = 0.9 \text{ p.u.}$ ,  $Q_o = 0.3 \text{ p.u.}$  The infinite bus voltage is  $V = 1 \text{ p.u.}$

Calculate for this operating condition:

- The per-unit terminal and excitation voltages.
- The power angle in degrees.
- The voltage regulation.
- The reluctance power in per-unit.
- The per-unit maximum power the generator can deliver without losing synchronism.
- $P_c$  and  $Q_c$  in per-unit.

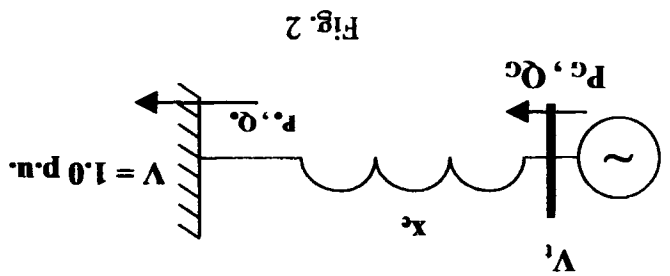
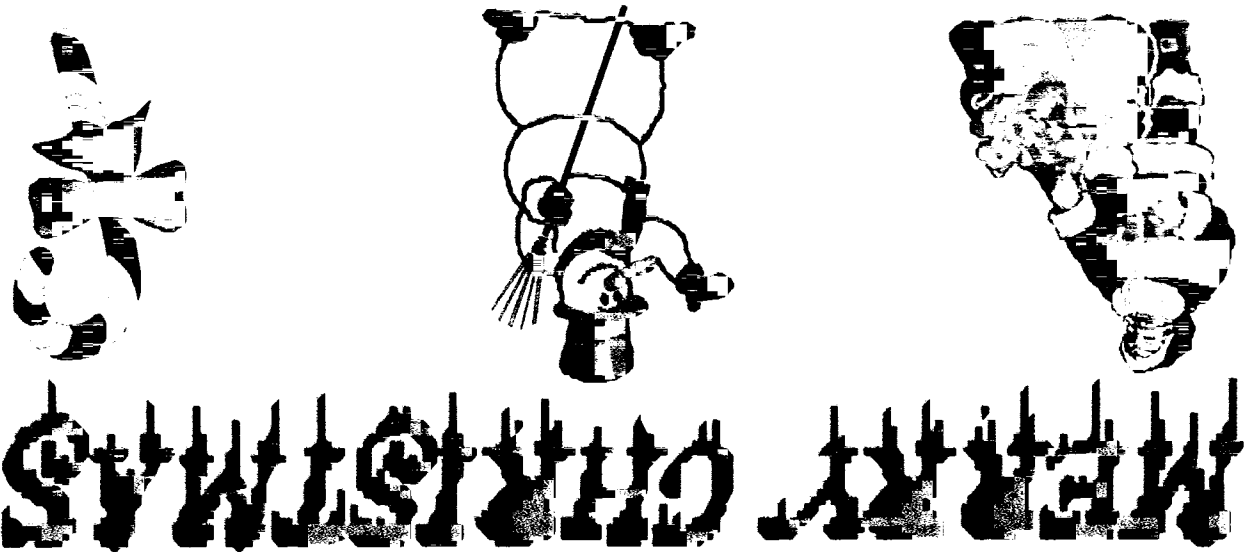


Fig. 2

6. A three-phase, Y-connected, round rotor synchronous motor has a synchronous reactance of 1.2 p.u. and negligible armature resistance.

- If the motor takes a line current of 0.9 p.u. at 0.85 p.f. leading from an infinite bus of 1.0 p.u. voltage, draw the vector diagram and calculate the excitation voltage and the power angle.

(b) If the motor is operating on load with a power angle of  $-30^\circ$ , and the excitation is adjusted that the excitation voltage is equal in magnitude to the terminal voltage, determine the active and reactive power delivered to the motor.



Instructor: S.O. Faried  
Duration: 3 Hours

December 16, 1997

1. (a) The torque expression of a three-phase induction motor can be given by:

$$T = \frac{3V_m^2 R_2' / s}{\omega_s [(R_m + R_2' / s)^2 + (X_m + X_2')^2]}$$

Show that in the limit of negligible armature resistance  $R_1$ , this expression can be written as

$$T = \frac{2T_{\max}}{\frac{s_{\max}}{s} + \frac{s}{s_{\max}}}$$

where  $T_{\max}$  is the maximum torque and  $s_{\max}$  is the slip at maximum torque.

- (b) A 230-V, 4-pole, 10-hp, 60-Hz, three-phase induction motor has the following per-phase equivalent circuit parameters:

$$\begin{aligned} R_1 &= 0.0 \, \Omega & R_2' &= 0.332 \, \Omega \\ X_1 &= 1.1 \, \Omega & X_2' &= 0.47 \, \Omega \\ X_m &= 26 \, \Omega \end{aligned}$$

- i) What is the maximum torque of this motor? At what speed and slip does it occur?  
ii) What is the starting torque of this motor?

2. A 100-MVA, 11.8 kV, 60-Hz, 2-pole, Y-Connected, synchronous generator has a per-unit synchronous reactance of 0.8 and a negligible armature resistance. The generator is connected to an infinite bus system of 1.0 p.u. voltage through a tie-line of 0.2 p.u. reactance.

- (a) If the generator is delivering its full-load current at 0.8 P.F. lagging to the infinite bus, find:

- i) the terminal voltage  $V_t$   
ii) the excitation voltage  $E_f$   
iii) the generator power angle  $\delta$   
iv) the voltage regulation.

- (b) If the generator excitation is adjusted such that the magnitude of the terminal voltage  $V_t$  is equal to the infinite bus voltage while the generator is still delivering its full-load current, draw the system vector diagram and find:

- i) the power factor at the infinite bus  
ii) the excitation voltage  $E_f$   
iii) the generator power angle  $\delta$   
iv) the maximum power that can be delivered without losing synchronism.

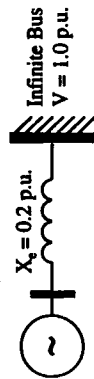


Figure 1

3. (a) Starting from the steady-state power-angle equation of a salient-pole synchronous machine with negligible armature resistance and fixed field excitation, show that the condition for maximum power is given by:

$$\cos \delta = -K + \sqrt{K^2 + 0.5}$$

where

$$K = \frac{E_f X_q}{4(X_d - X_q)V}$$

- (b) The direct-and quadrature-axis synchronous reactances of a salient-pole synchronous generator are  $X_d = 1.0$  p.u. and  $X_q = 0.8$  p.u. The generator is connected to an infinite bus of 1.0 p.u. voltage.

- i) If the machine loses synchronism when the power angle is  $81.414473^\circ$ , what is the p.u. excitation voltage at pullout?  
ii) For the case described in (i), what are the corresponding active and reactive powers?

4. In the two-machine system shown in Figure 2, the excitations of the two machines are so controlled that the terminal voltages of the two machines remain constant and equal to 1.0 p.u.

- (a) Derive an expression for the power fed from the synchronous generator to the synchronous motor as a function of their terminal voltages  $V_g$  and  $V_m$  and the angle between the quadrature axes of the two machines, ( $\delta$ ).

- (b) What will be the maximum power which can be fed without losing synchronism?
- (c) What is the value of  $\delta$  in this case?

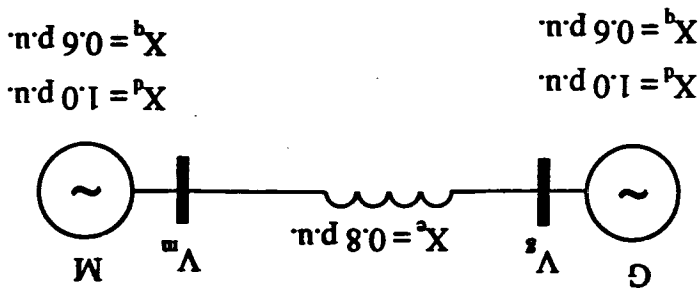


Figure 2

5. In the system shown in Fig. 3, one circuit of the double-circuit transmission line is opened suddenly. The system parameters and operating conditions before the disturbance are indicated in the same figure. Using the equal-area criterion, check the transient stability of the system after this disturbance. If it is stable, find the maximum angle of swing.

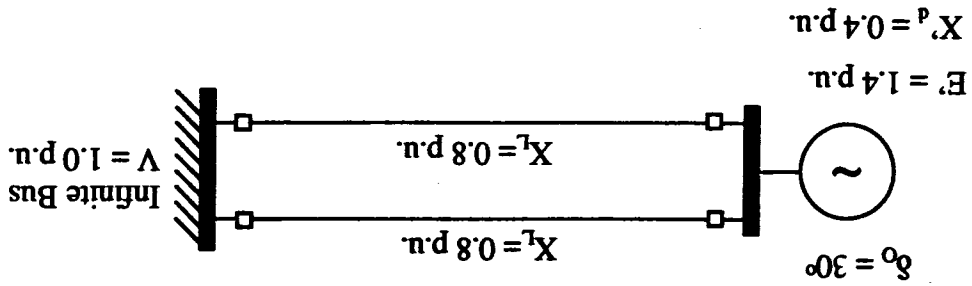


Figure 3

... The End ...

11/15  
EE461 Midterm

(2) 2. Find the difference equation for a system that has output

$$y(n) = 0.25^n(u(n) + 0.75u(n-1)) + 0.75^n u(n)$$

when the input is

$x(n) = 0.25^n u(n)$  [in assuming this is  $u(n)$  so that I can complete the question (even though it will be wrong)]

$$\begin{aligned} y[n] &= 0.25^n u[n] + 0.75 \cdot 0.25^n u[n-1] + 0.75^n u[n] \\ Y(z) &= \frac{1}{1-0.25z^{-1}} + \frac{0.75X}{1-0.25z^{-1}} + \frac{1}{1-0.75z^{-1}} \\ &= \frac{1.75(1-0.75z^{-1}) + 1-0.25z^{-1}}{(1-0.25z^{-1})(1-0.75z^{-1})} = \frac{1.75 + 1 + 1.3z^{-1} - 0.25z^{-1}}{(1-0.25z^{-1})(1-0.75z^{-1})} \\ &= \frac{2.75 + 1.06z^{-1}}{(1-0.25z^{-1})(1-0.75z^{-1})} \\ X[n] &= 0.25^n u[n] \\ X(z) &= \frac{1}{1-0.25z^{-1}} \end{aligned}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2.75 + 1.06z^{-1}}{(1-0.25z^{-1})(1-0.75z^{-1})} = \frac{2.75 + 1.06z^{-1}}{1-0.75z^{-1}}$$

$$Y(z)(1-0.25z^{-1}) = X(z)(2.75 + 1.06z^{-1})$$

$$y[n] - 0.25y[n-1] = 2.75x[n] + 1.06x[n-1]$$

Date: Wednesday, October 9, 2002

Time = 1 hour 30 minutes

Test Books and Notes Only - no worked examples or solved problems

1. An engineer is to design a NCO that has a frequency resolution of less than  $10^{-6}$  radians/sample (i.e. the frequency can be incremented in steps of  $\Delta\omega$ , where  $\Delta\omega < 10^{-6}$  radians/sample) and an SNR of greater than 50 dB on the output sinusoid.

- (2) (a) What is the minimum size that can be used for the phase accumulator?
- (3) (b) What is the minimum size ROM (LUT) that can be used? Specify the size in number of bits.

a)  $\Delta\omega < 10^{-6}$  rad/sample, SNR > 50dB

$\Delta F < 1.6 \times 10^{-6}$  cycles/sample

The number of bits in the P.A.,  $N_1$ , should obey:

$$\frac{1}{2^N} < 1.6 \times 10^{-6} \text{ cycles/sample}$$

for  $N = 20, \frac{1}{2^{20}} = 9.54 \times 10^{-7}$ , so select  $N = 20$

b) Find  $N_0$  and  $N_A$  such that SNR > 50dB

$N_0$	$N_A$	SNR
11	11	60.25 dB
10	11	58.48 dB
10	10	54.23 dB
9	10	52.46 dB
9	9	48.21 dB
10	9	48.80 dB
9	10	48.76 dB

\* 9 10 10 52.46 dB ← Best Combination corresponds to optimal of  $N_A = N_0 + 1$

total # of bits in Rom = # addresses × # bits/address

$= 2^9 \cdot 9$

$= 9261 \text{ bits}$



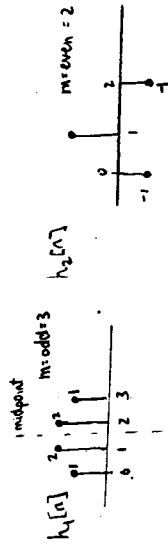
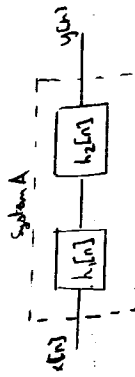
- (2) 4. A system, say system A, is composed of two systems in tandem (cascade). The two systems in tandem (cascade) have impulse responses

$$h_1(n) = \delta(n) + 2\delta(n-1) + 2\delta(n-2) + \delta(n-3)$$

and

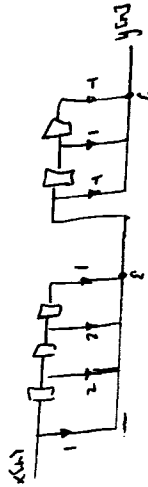
$$h_2(n) = -\delta(n) + \delta(n-1) - \delta(n-2)$$

Find an expression for the phase response of system A. (i.e. find  $\angle H_A(e^{j\omega})$ )



- both responses are symmetric.  
- both subsystems are FIR

- System A will have a Symmetric Response



$h_1[n]$  is a linear phase system:  $H_1(e^{j\omega}) = H_1(e^{j\omega})$

$$H_1(e^{j\omega}) = e^{-j\frac{3}{2}\omega} A(\omega) \times H_2(e^{j\omega})$$

$$\angle H_1(e^{j\omega}) = \angle e^{-j\frac{3}{2}\omega} = -\frac{3}{2}\omega$$

$h_2[n]$  is a linear phase system:  $H_2(e^{j\omega}) = H_2(e^{j\omega})$

$$\angle H_2(e^{j\omega}) = \angle e^{-j\omega} = -\omega$$

$$\angle H_A(e^{j\omega}) = e^{-j\frac{3}{2}\omega} \times e^{-j\omega} = -\left(\frac{3}{2}\omega + \omega\right)$$

- (2) 3. Find the impulse response if the system function is

$$H(z) = \frac{1+j1}{1-j0.5z^{-1}} + \frac{1-j1}{1+j0.5z^{-1}}$$

- need to find an acceptable form to take inverse z-transform

$$H(z) = \frac{(1+j)(1+j0.5z^{-1}) + (1-j)(1-j0.5z^{-1})}{(1-j0.5z^{-1})(1+j0.5z^{-1})}$$

$$= \frac{1+j(1+j0.5z^{-1}) - 0.5z^{-1} + 1-j(1-j0.5z^{-1}) - 0.25z^{-1}}{1 - (j0.5z^{-1} + j0.5z^{-1}) + 0.25z^{-1}} = \frac{2 - 1z^{-1}}{1 + 0.25z^{-1}}$$

$$H(z) = \frac{2}{1+0.25z^{-1}} = \frac{2}{1+0.25z^{-1}} \cdot \frac{z}{z} = \frac{2z}{z+0.25}$$

$$h[n] = 2(-0.25)^n u[n] - \delta[n-1] \quad \text{delay of 1 sample}$$

$$h[n] = 2(-0.25)^n u[n] - (-0.25)^{n-1} u[n-1]$$

# EE461 Midterm

NAME: [REDACTED]

STUDENT NO.: [REDACTED]

Date: Wednesday, November 20, 2002

Time = 1 hour 30 minutes

Text Books and Notes Only

Absolutely no worked examples or solved problems

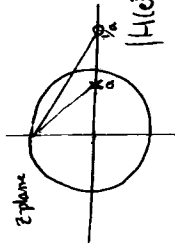
1.	4
2.	5
3.	3 1/2
4.	5
5.	5
6.	1
TOTAL	23 1/2

5. Consider a causal linear time-invariant system with system function

$$H(z) = \frac{1 - a^{-1}z^{-1}}{1 - az^{-1}}$$

- (1) (a) What is  $|H(e^{j\omega})|$  at frequencies  $\omega = 0$ ,  $\omega = \pi/2$ , and  $\omega = 7\pi/8$  radians per sample?
- (2) (b) Write the difference equation that relates the input and the output of the system.
- (1) (c) for what range of  $a$  is the system stable?

For a real system, pole must be inside unit circle implying that the zero is closer to origin



$$\begin{aligned} |H(e^{j0})| &= \frac{1-a^{-1}}{1-a} \\ |H(e^{j\pi/2})| &= \frac{\sqrt{\frac{1}{a^2} + 1}}{\sqrt{a^2 + 1}} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{graphical method}$$

$$|H(e^{j7\pi/8})| = \left| \frac{1 - \frac{1}{a} e^{-j7\pi/8}}{1 - a e^{j7\pi/8}} \right|$$

$$\begin{aligned} &= \left| \frac{1 - \frac{1}{a} (\cos 7\pi/8 - j \sin 7\pi/8)}{1 - a (\cos 7\pi/8 + j \sin 7\pi/8)} \right| \\ &= \left| \frac{1 - \frac{1}{a} \cos 7\pi/8 + \frac{1}{a} j \sin 7\pi/8}{1 - a \cos 7\pi/8 - a j \sin 7\pi/8} \right| \end{aligned}$$

= ?

$$b) H(z) = \frac{1 - a^{-1}z^{-1}}{1 - az^{-1}} = \frac{X(z)}{Y(z)} \quad X(z) = Y(z)$$

$$Y(z) - Y(z)a^{-1}z^{-1} = X(z) - X(z)a^{-1}z^{-1}$$

$$Y(z)[1 - a^{-1}z^{-1}] = X(z)[1 - a^{-1}z^{-1}]$$

c) the pole must be inside the Unit Circle

$$0 < |a| < 1$$

this represents an all pass system

(5)

2. A system has a finite impulse response of length 5 (i.e.  $M=4$ ). When an input of  $\sqrt{2} \cos(\frac{\pi}{4}n)$  is applied, the output for  $n = 0, 1, \dots, 5$ , is the real sequence  $\{1.4, 3.8, -12.1, -6.8, -42.84, -23.828\}$ . When an input of  $\sqrt{2} \sin(\frac{\pi}{4}n)$  is applied, the output for  $n = 0, 1, \dots, 5$ , is the real sequence  $\{0, 1, 3.4, -6.2, -9.142, -36.757\}$ . What is the frequency response of the system at  $\omega = \pi/4$  radians per sample?

$$H(e^{j\omega}) = y[5]e^{-j5\pi/4} \quad \text{for } x[n] = e^{j\pi/4}$$

$$x[n] = \sqrt{2} \cos(\pi/4 n)$$

$$x[n] = \sqrt{2} \sin(\pi/4 n)$$

$$\text{for } x[n] = e^{j\pi/4}$$

$$\text{then } x[n] = (x[n] + jx[n]) \frac{1}{\sqrt{2}}$$

$$= \{1, 2.7 + j0.707, -8.56 + j2.404 + 808 - j4.26, -303 - j6.46, -16.85 - j26\}$$

$$H(e^{j\pi/4}) = (-16.85 - j26) e^{-j5\pi/4}$$

$$|H(e^{j\pi/4})| = 31 \angle -0.567\pi$$

$$H(e^{j\pi/4}) = y[4]e^{-j2\pi} = y[4]$$

$$\begin{aligned} H(e^{j\pi/4}) &= -42.84/\sqrt{2} + j23.828/\sqrt{2} \\ &= 31e^{j-2.93} \\ &= 31 \angle -168^\circ \end{aligned}$$

(7)

1. Please circle the correct answer for the questions that follow. Note that wrong answers will be subtracted from the right answers. All parts are worth the same.

The questions are based on three discrete time systems, each with system functions containing only zeros. System 1 has 6 zeros located at  $z = 0.7e^{j0.9}, 0.7e^{-j0.9}, 1, -1, .5$  and 2. System 2 has 22 zeros at  $z = c_k$ , where  $c_k = e^{j\frac{2\pi k}{23}}$ ,  $k = 2, 3, \dots, 23$ . System 3 has 17 zeros, with 4 at  $z = 1, 3$  at  $z = -1, 5$  at  $z = e^{j\pi/4}$  and 5 at  $z = e^{-j\pi/4}$ .

- (a) The impulse response of system 1 is  
 a) symmetric, b) antisymmetric, c) neither symmetric nor antisymmetric about its midpoint.
- (b) The impulse response of system 2 is  
 a) symmetric, b) antisymmetric, c) neither symmetric nor antisymmetric about its midpoint.
- (c) The impulse response of system 3 is  
 a) symmetric, b) antisymmetric, c) neither symmetric nor antisymmetric about its midpoint.
- (d) The magnitude of the frequency response of system 3 is greater at  
 a)  $\omega = \pi/4$  radians/sample or b)  $\omega = 3\pi/4$  radians/sample.
- (e) The magnitude of the frequency response of system 2 is  
 a) zero b) not zero at  $\omega = \pi$  radians/sample.
- (f) The magnitude of the frequency response of system 1 is  
 a) zero b) not zero at  $\omega = 0.5$  radians/sample.
- (g) The phase of the frequency response of system 2 at  $\omega = \pi/10$  radians per sample (i.e. angle of  $H_2(e^{j\pi/10})$  is  
 a)  $-17\pi/20$  radians b)  $-27\pi/20$  radians c) neither a) nor b)

4. A digital filter is constructed by sampling the impulse response of an analog filter with a sampling rate of 1000 samples/second. Find an expression for the frequency response of the digital filter if the analog filter has system function

$$H_a(s) = \frac{s+7}{(s+3)(s+2)}$$

$$F = 1000 \text{ samp/sec} \quad T_d = \frac{1}{1000} \text{ sec/sample}$$

→ Sampling impulse response is impulse invariant

$$H_d(z) = \frac{A}{s+3} + \frac{B}{s+2}$$

$$A = \frac{s+7}{s+2} \Big|_{s=-3} = \frac{4}{-1} = -4$$

$$B = \frac{s+7}{s+3} \Big|_{s=-2} = \frac{5}{1} = 5$$

$$H_d(s) = \frac{5}{s+2} - \frac{4}{s+3}$$

→ assuming  $h[n] = T_d h_a(nT_d)$

$$\text{then; } H(z) = \frac{T_d 5}{1 - e^{-3T_d} z^{-1}} - \frac{T_d 4}{1 - e^{-2T_d} z^{-1}}$$

$$H(z) = T_d \left( \frac{5(1 - e^{-3T_d} z^{-1}) - 4(1 - e^{-2T_d} z^{-1})}{1 - (e^{-3T_d} + e^{-2T_d})z^{-1} + e^{-(2T_d+3T_d)}z^{-2}} \right)$$

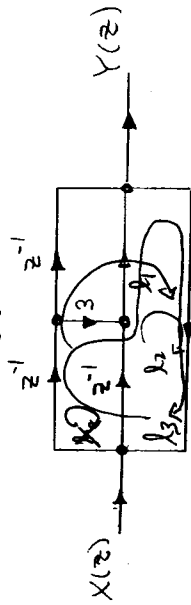
$$H(z) = T_d \left( \frac{1 - 5e^{-3T_d} z^{-1} + 4e^{-2T_d} z^{-1}}{1 - (e^{-3T_d} + e^{-2T_d})z^{-1} + e^{-(2T_d+3T_d)}z^{-2}} \right)$$

$$H(z) = 0.001 \left( \frac{1 + (4e^{-0.002} - 5e^{-0.003})z^{-1}}{1 - (e^{-0.002} + e^{-0.003})z^{-1} + e^{-0.005}z^{-2}} \right)$$

$$H(e^{j\omega}) = 0.001 \left( \frac{1 + (4e^{-0.002} - 5e^{-0.003})e^{-j\omega}}{1 - (e^{-0.002} + e^{-0.003})e^{-j\omega} + e^{-0.005}e^{-j2\omega}} \right)$$

$$H(e^{j\omega}) \approx 0.001 \left( \frac{1 - 1e^{-j\omega}}{1 - 2e^{-j\omega} + e^{-j2\omega}} \right)$$

3. Redraw the graph below in direct form 2 structure. Show all the coefficients on the direct form 2 graph.



Simplified:

$$P_1 = z^{-1} \quad l_1 = z^{-2}$$

$$P_2 = z^{-2} \quad l_2 = z^{-1}$$

$$P_3 = 3z^{-1} \quad l_3 = 3z^{-1}$$

$$\Delta = 1 - (3z^{-1} + z^{-1})z^{-2}$$

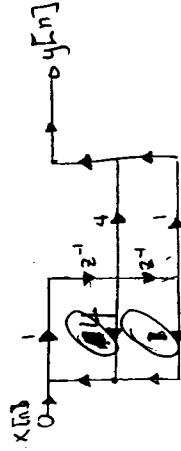
$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

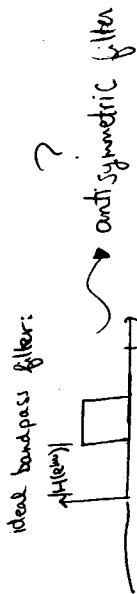
$$\Delta_3 = 1$$

$$H(z) = \frac{\sum P_i \Delta_i}{\Delta} = \frac{z^{-1} + z^{-2} + 3z^{-1}}{1 - 2z^{-1} - 3z^{-2}}$$

$$H(z) = \frac{4z^{-1} + z^{-2}}{1 - 2z^{-1} - 3z^{-2}}$$



6. Find an expression for the coefficients,  $b_k$ ,  $k = 0, 1, \dots, M$ , for a symmetric linear phase filter of length  $M+1$ , where  $M$  is even, that best approximates an ideal bandpass magnitude response, with the pass band between  $\omega_L$  and  $\omega_U$ .



$$|H(e^jw)| = \begin{cases} 1, & \omega_L < \omega < \omega_U \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \frac{1}{2\pi} \int_{\omega_L}^{\omega_U} \sin(\omega(n-m)) d\omega$$

$$h[n] = \begin{cases} 0 & n = M/2 \\ \frac{1}{2\pi} \cdot \frac{1}{(n-M/2)} [\cos(\omega_U(n-M/2)) - \cos(\omega_L(n-M/2))] & n \neq M/2 \end{cases}$$

5. A digital filter is constructed by a bilinear transformation on an analog filter with a sampling rate of 1000 samples/second. Find an expression for the frequency response of the digital filter if the analog filter has system function

$$H_a(s) = \frac{s+7}{(s+3)(s+2)}$$

$$H(z) = H_a(s) \Big|_{s = \frac{2}{T} \left( \frac{1-z^{-1}}{1+z^{-1}} \right)}$$

$T = \frac{1}{1000} \text{ sample}$

$$H(z) = \frac{\frac{2}{T} \left( \frac{1-z^{-1}}{1+z^{-1}} \right) + 7}{\left[ \frac{\frac{2}{T} \left( \frac{1-z^{-1}}{1+z^{-1}} \right)^2 + 5 \left[ \frac{2}{T} \left( \frac{1-z^{-1}}{1+z^{-1}} \right) \right] + 6}{\left[ T(1+z^{-1}) \right]^2} \right]}$$

$$= \frac{2(1-z^{-1})[T(1+z^{-1})] + 7[T(1+z^{-1})]^2}{[2(1-z^{-1})]^2 + 5[2(1+z^{-1})][T(1+z^{-1})] + 6[T(1+z^{-1})]^2}$$

$$= \frac{2T(1-z^{-2}) + 7T^2(1+z^{-1}+z^{-2})}{4(1-2z^{-1}+z^{-2}) + 10T(1-z^{-2}) + 6T^2(1+2z^{-1}+z^{-2})}$$

$$= \frac{(2T+7T^2) + 14T^2z^{-1} + (7T^2-2T)z^{-2}}{(4+10T+6T^2) + (12T^2-8)z^{-1} + (6T^2+10T+4)z^{-2}}$$

$$H(e^{j\omega}) = \frac{(2T+7T^2) + (14T^2)e^{-j\omega} + (7T^2-2T)e^{-j2\omega}}{(4+10T+6T^2) + (12T^2-8)e^{-j\omega} + (6T^2+10T+4)e^{-j2\omega}}$$

with  $T=0.001$

$$H(e^{j\omega}) = \frac{0.002007 + 0.000014 e^{-j\omega} - 0.001993 e^{-j2\omega}}{4.000006 - 7.999999 e^{-j\omega} + 3.999996 e^{-j2\omega}}$$

# EE484 MIDTERM 2

Thursday, March 22, 2001

Time - 1 hour.

Only two formula sheets allowed.

All Questions worth 5

1. A bilinear transformation is used to transform continuous-time system function

$$H_c(s) = \frac{0.02}{s^2 + 0.2s + 0.02}$$

to discrete-time system function  $H(z)$ .

- (a) Find the poles and zeros of  $H(z)$ . (NOTE: Be careful as the answers to parts b) and c) depend on this answer being correct.)
- (b) Is this a low-pass, band-pass or high-pass filter? (To obtain credit you must justify your answer.)
- (c) Is there ripple in the stopband? (To obtain credit you must justify your answer.)

2. An junior engineer is asked to design a digital band-pass filter by applying a bilinear transformation to an analog band-pass filter. The digital filter is specified as follows:

$$1 - 0.1 < |H(e^{j\omega})| < 1 + 0.1; \quad 0 \leq \omega < \frac{\pi}{4}$$

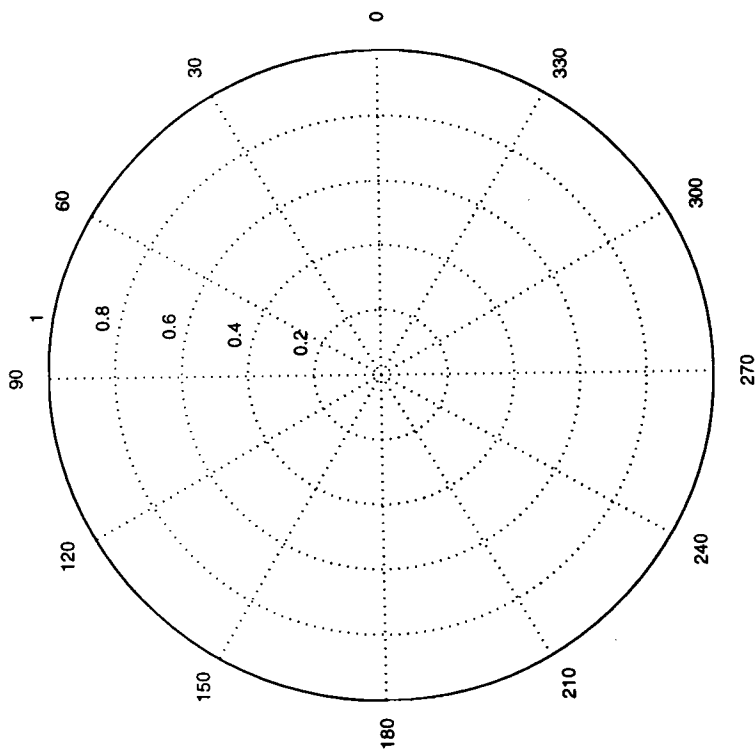
$$|H(e^{j\omega})| < 0.01; \quad \frac{\pi}{2} \leq \omega \leq \pi$$

Specify the analog filter that has to be designed.

3. Find the order and parameter  $\Omega_c$  for a low pass Butterworth filter that satisfies:

$$0.9 \leq H_c(j\Omega) \leq 1; \quad 0 \leq \Omega \leq \frac{\pi}{4}$$

$$H_c(j\Omega) \leq 0.01; \quad \frac{\pi}{2} \leq \Omega \leq \pi\omega$$



From  
mid #1  
2001

- Determine the order of the filter.
- Determine the impulse response, the transfer function and the difference equation for the filter.

c) Is this an IIR or FIR filter? Is this a lowpass/highpass/bandpass or bandstop filter?

d) What are the addresses at which the D/A and A/D converter are found? Determine the number of bits of resolution for A/D and D/A converter and their position in the D/A and A/D register.

e) Lines 10 and 11 are replaced as follows

```
10 LDI    0HEX3,AR1      ;
11 LDI    0HEX4,AR0 ;
```

Determine appropriate values for lines 4,5,20,32,33. If an input sequence given below is applied

$$x(n) = \{1, 5, 2, 3, 4, 0, 0, 0, 0\}$$

(1)

Determine the output of the filter to this input sequence. What type if filter is implemented by this code?

f) What is the gain of the filter? Show how to change the gain of the filter to 2.0.

g) Add a 1V DC offset to the filtered output by modifying line 20. Assume a  $\pm 8V$  output range for the D/A converter.

## 2 Question #2

The following output was generated by Matlab

```

f=[0 1 1 2 3 36 4 8 6 7 8 9 1]
%
Columns 1 through 7
a =
0 0.1000
Columns 8 through 12
0.0000 0.7000 0.8000 0.9000 1.0000
m=[0 0 0 1 1 1 0 0 0]
%
D=[a m pulesm3(0,f,a)]
%
b =
0 1 1 1 1 1 0 0 0 0
%
a =
0.1104 - 0.0045 - 0.1815 0.0009 0.1822 0.0041 -0.1101
%
[x,p,k] = ssmp(b,a)
%
s =
-0.0007 + 0.0094i
-0.0007 - 0.0094i
0.0008 + 0.0001i
0.0008 - 0.0001i
0.0258 - 0.0094i
-0.0093
0.0043
%
p =
0.0007 + 0.0094i
0.0007 - 0.0094i
-0.0008 + 0.0001i
-0.0008 - 0.0001i
-0.0258 - 0.0094i
-0.0094 + 0.0094i
0.0258 + 0.0094i
%
z =
0.1104

```

- Is this an IIR or FIR filter? Is this a lowpass/highpass/bandpass or bandstop filter?
- Determine the order, the transfer function and the difference equation for this filter.
- Plot the poles and zeros of the filter.

**The following is another set of output from Matlab**

[illegible][illegible]

- e) Is this an IIR or FIR filter? Is this a lowpass/highpass/bandpass or bandstop filter? What is the order of this filter?
- f) If the sampling frequency is  $34\text{ kHz}$ , determine the cutoff frequency.
- g) Plot the magnitude and phase response of the filter.
- h) Determine the transfer function and difference equation of this filter.



### 3 Question #3

Discovered in the basement archives of a Nashville recording studio is an unreleased original, early recording by Elvis. It seems as if the recording was discarded due to significant corruption. The recording is corrupted by harmonic distortion that is given by

$$D(\omega) = 0.5^k \cos(2\pi f_0 k)$$

for  $f_0 = 1\text{KHz}$  and  $k = 1, 2, 3, 4, 5, 6$ .

- Design a comb filter that will remove this distortion. Specify the transfer function, difference equation and sampling rate.
- After digitally processing the recording, it was played for a studio executive, who was not satisfied with the results. Further analysis indicates that a cascade of three notch filters, to remove the first three harmonics, will provide better results. The sampling rate is specified as 16KHz. Each of the notch filters is to have a 3db bandwidth of 50Hz. Determine the transfer function and difference equation for the notch filter that will remove the 1KHz distortion. Assume each notch filter can be designed independently.

6

DO ANY TWO OF THE FOLLOWING FOUR QUESTIONS  
IE Answer any two questions out of questions 4,5,6 and 7.

### 4 Question #4

Design a Lowpass filter using the Frequency Sampling Method.

- Determine the coefficients of a linear-phase FIR filter of length  $M = 15$  which has a symmetric unit sample response and a frequency response that satisfies:

$$H_d\left(\frac{2\pi k}{15}\right) = \begin{cases} 1 & k = 0, 1, 2, 3, 4 \\ 0.3927 & k = 5 \\ 0 & k = 6, 7 \end{cases}$$

- Plot the magnitude and phase for the above filter at  $\omega = \{0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{2}\}$

### 5 Question #5

- Design an FIR linear-phase digital filter that has the following approximate frequency response

$$H_d(\omega) = \begin{cases} 1 & \text{for } |\omega| \leq \frac{\pi}{3} \\ 0 & \text{for } \frac{\pi}{3} < |\omega| \leq \pi \end{cases}$$

Determine the coefficients for a 6th order filter based upon a Hanning window.

- For the above filter, determine the gain value  $K$ , such that gain of the filter is unity (ie 1).

7

## 6 Question #6

A researcher in the Dept. of Biology has designed an experiment to investigate the effect of temperature on the number of ducklings hatched from a nest. Under each nest he has placed a temperature probe and he has decided to sample the temperature once per minute (assume no aliasing). Further more he has decided to average the present temperature reading with the past three readings to create a filtered temperature value,  $y(n) = \frac{1}{4}(x(n) + x(n-1) + x(n-2) + x(n-3))$ .

a) Given the implementation of his data acquisition and filtering, which periodic temperature fluctuations in his experiment will be eliminated and hence perhaps adversely affect his experimental results?

## 7 Question #7

Given the following transfer function

$$H(z) = \frac{0.2248 + 0.3299z^{-1} + 0.2388z^{-2}}{1.0 - 0.4601z^{-1} + 0.2388z^{-2}}$$

- Determine the poles and zeros and plot in the Z-plane.
- Sketch the magnitude response at  $\omega = \{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\}$
- Show a Direct Form II realization.
- Show a Direct Form I realization of this filter.

February, 1998

Time: 50 minutes

Textbook, Notes and Calculators Allowed

A casual filter is described by

$$H(z) = b_o \left[ \frac{1 - 2b \cos(\frac{\pi}{4}) z^{-1} + b^2 z^{-2}}{1 - 2a \cos(\frac{\pi}{4}) z^{-1} + a^2 z^{-2}} \right]$$

$$b = 0.95 ; a = 0.99$$

- Sketch the pole-zero pattern for this filter in the z-plane. Be sure to show the unit circle.
- From the pole-zero plot, sketch the magnitude response of the filter.
- From the pole-zero plot, sketch the phase response of the filter.
- Determine  $b_o$  so that the maximum gain is approximately 1.
- Show the direct form I and direct form II realizations of this filter. Be sure to specify all coefficients.
- What type of filter is this and what is the approximate bandwidth?
- Determine a new set of coefficients for the direct form I realization that will approximately double the bandwidth while keeping the ratio of pass-band to stop-band gain nearly the same.

Tuesday, March 3, 1998

Time: 50 minutes

Textbook, Notes and Calculators Allowed

- If the following systems are not already minimum phase systems, convert them to minimum phase systems without changing the magnitude response and give the impulse response of the new system.

a)  $h(n) = [1 \quad -4 \quad 3]$

↑

b)  $h(n) = [-1 \quad 4 \quad -4]$

↑

- Determine the minimum-phase system whose magnitude squared response is:

$$|H(\omega)|^2 = 101 + 10e^{j\omega} + 10e^{-j\omega}$$

- Design a single pole, single zero, high pass filter with cutoff frequency  $\frac{19\pi}{20}$ .

April 1997

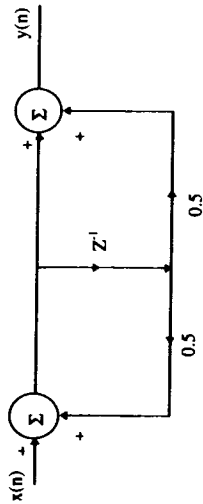
Instructor: J.E. Salt

Time: 3 Hours

Note: Textbook and notes allowed

**Marks**

1. Consider the filter below.



- (4) (a) Plot the pole zero pattern.

- (4) (b) What is  $y(n)$  if

i)  $x(n) = \cos \frac{\pi}{2} n$

ii)  $x(n) = \cos \pi n$

- (8) (c) Specify the resolution of the adders and multipliers (as well as the amount of truncation) needed to implement the filter in an application specific integrated circuit. The input is quantized and represented in 8 bit, two's complement format.

- (20) 2. Draw a flow graph of the filter implemented in the TMS320C31 assembler code shown below. Be sure to show the value and sign of all the coefficients. Also be sure to mark the inputs to a summer with a minus sign if you wish to subtract.

- include "initial.asm"

```

X_ADDR .SET 808048H
R_ADDR .SET 80804CH

.sect ".text"

MAIN: LDI 3, BK
      LDI @ BUFF_AD, ARO
      LDI @ COEF_AD, ARI

      WAIT B WAIT

ISR: LDF 0, R0
     LDF 0, R2
     RPTS 2
     MPYF3 *ARO++, *ARI++, R0
     || ADDEF3 R0, R2, R2
     ADDEF3 R0, R2, R2
     LDI @R_ADDR, R0
     LSH 16, R0
     ASH -18, R0
     FLOAT R0, R0
     ADDEF3 R0, R2, R2
     STF R2, *ARO++
     FIX R2, R2
     LSH 2, R2
     STI R2, @X_ADDR
     RETI

BUFF_AD .word 809900H
COEF_AD .word 809A00H

.start "flt_coef", 809A00H
.sect "flt_coef"
.float 0.1
.float 0.2
.float 0.3
.float 0.4
.float 0.5
.float 0.6

```

```
.start "serveet", 809FCSH
.sect "set vect"
```

RET I  
B ISR

3. A filter was designed using the frequency sampling technique with the following matlab code. Two trials were done. A second frequency response statement was added after the program was run with the first frequency response statement. The matlab output for the two runs is shown after the code.

(20)

- Is the filter a linear phase filter and if so what type of linear phase filter is it.
- Plot the two impulse responses obtained from the two trials.
- Plot the two frequency responses you would expect from the two specifications.
- Plot the two phase responses as well.
- What the is bandwidth of the filter?

```
%parameters
N = 11; % filter length
w=[0.1*pi .3*pi .5*pi .8*pi pi];
A_w = [1 1 0 0 0]; %desired magnitude response at frequencies in w
A_w = [1 .95 .5 1 0 0];
% calculation of the cosine matrix
n = [0:(N-1)/2];
cos_matrix =cos(w.*(n-(N-1)/2));
% find the impulse response
two_H = inv(cos_matrix)* A_w.;
H = two_H/2;
H((N-3)/2)=2*H((N-3)/2); % last element of two_H was not double so fix it now
H = H
```

MATLAB COMMAND WINDOW

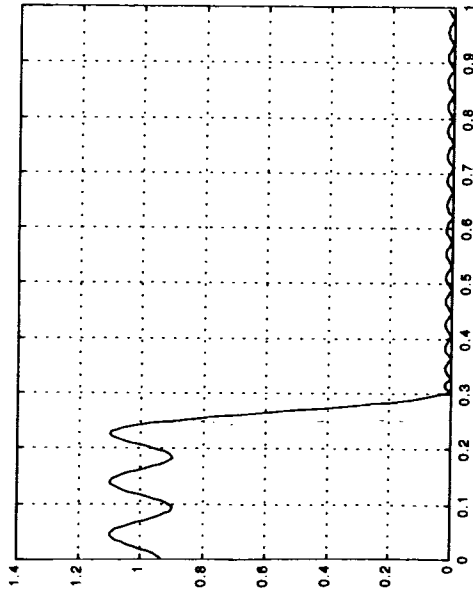
```
>> final_98_question
```

```
H =
-0.1101
-0.0068
0.2016
0.2500
0.1584
0.1318
```

```
>> final_98_question
```

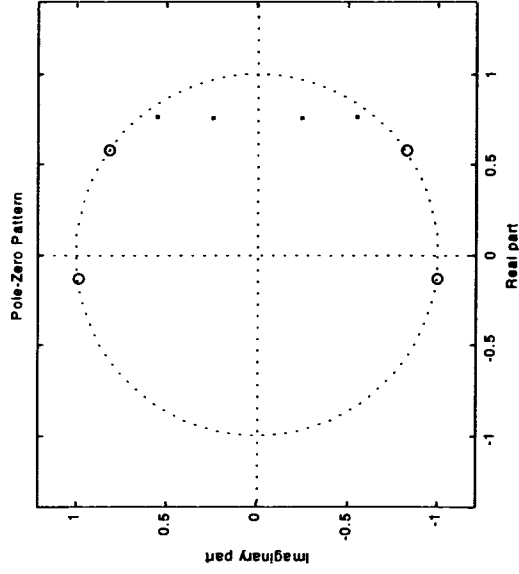
```
H =
-0.0100
-0.0075
0.0231
0.2000
0.2369
0.1575
```

- 4) a) What matlab commands were used to obtain the filter response shown below?  
(12) b) What is the approximate order of the filter?  
c) Are there any zeros located on the real axis. If so, state there approximate location? Be sure to explain your reasoning .



5) The pole-zero pattern for a low-pass filter is shown below.

- What is the filter type?
- What is the approximate stop band attenuation?
- What is the approximate pass-band corner frequency?



6. (a) Find the DFT for the sequence

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

↑

(20)

(b) Find the DFT of the  $N$  samples from  $n = 0$  to  $n = N-1$  of the sequence  $x(n) = a^{2n}$ .

(c) Find the inverse DFT of

$$X(k) = [0 \quad -j \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad j]$$

(d) Find the DFT of

$$x_3(n) = x_1(n) \odot_N x_2(n)$$

for  $x_1(n) = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$

$$x_2(n) = [0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6]$$

**Time:** 3 hours.

**Instructor:** Prof. J.E. Salt

**Note:** Text and notes allowed

April 1995

**Marks**

- (10) 1. Find the discrete time Fourier Transform of

$$x(n) = \begin{cases} a^n & \text{for } n \text{ even; } n \geq 0 \\ b^n & \text{for } n \text{ odd; } n \geq 1 \end{cases}$$

- (10) 2. Find the discrete time Fourier Transform of  $Y(\omega)$  in terms of  $X(\omega)$  if  $y(n)$  is related to  $x(n)$  by

$$(a) \quad y(n) = \left( \sum_{k=-\infty}^{\infty} x(k)x(n-k) \right) \cos \omega_0 n$$

where  $\omega_0$  is a constant.

$$(b) \quad y(n) = x^*(n-1)e^{j\pi/2}$$

- (5) 3. (a) Find the steady state response of the system with impulse response

$$h(n) = \left(\frac{1}{4}\right)^n u(n-3)$$

$$\text{if the input is } x(n) = \cos \frac{\pi}{3} n \quad Y(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

- (5) (b) The steady state output of a system when the input is  $x(n) = \cos \omega_0 n$  is

$$y_{ss}(n) = \left| \frac{1}{1 - 0.9e^{-j\omega_0}} \right| \cos(\omega_0 n + \theta(\omega_0)) \text{ for any } \omega_0,$$

$$\text{where } \theta(\omega_0) = -3\omega_0 - \angle(1 - 0.9e^{-j\omega_0}).$$

What is the frequency response of the system?

.../2

April 1995

**Marks**

- (5) 4. (a) Find the Z transforms of:

$$x(n) = \alpha^{2n} u(n) + \delta(n+10)$$

- (5) (b) Find the Z transform of  $y(n)$  in terms of the Z transform of  $x(n)$  if  $y(n)$  is related to  $x(n)$  by

$$y(n) = n x(-n).$$

The region of convergent of  $X(z)$  is  $r_1 < |z| < r_2$ .

- (5) 5. Prove that

$$\sum_{n=0}^{N-1} (\cos \omega_0 n + \sin \omega_0 n)^2 = N$$

for  $\omega_0 = \frac{\pi k}{N}$  for any integer  $k$ .

- (10) 6. Give the block diagram of a filter (showing all delays, sums and multiplier coefficients) that has a single pole at  $z = 0.5$  and a double zero at  $z = 1$ . The gain of the filter at  $\omega = \pi$  is 4.

- (5) 7. Find the inverse z transform of the stable system

$$X(z) = \frac{7z^2}{(z - \frac{1}{4})(z - 2)}$$

- (5) 8. (a) Is it possible to get a low pass filter with the 3dB down point at  $\omega = \frac{\pi}{4}$  and a

$$\text{relative gain } \left| \frac{H(\pi)}{H(0)} \right| = .2 \text{ with a single pole filter?}$$

If it is possible, give the location of the pole.

If it is not possible, either prove it or carefully explain it.

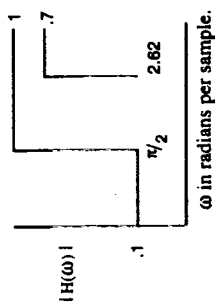
- (10) (b) Design a notch filter to remove the 60Hz component of a signal. The gain of the filter must be between .95 and 1 for all frequencies except those within 5 Hz of 60Hz. The sampling rate of the system is 2400 Hz.

.../3

April 1995

**Marks**

- (15) (c) Design a high-pass filter to the template given below.



- (6) 9. Classify the following system functions as linear or non linear phase filters?  
(A wrong answer will result in negative marks).

(a)  $H(z) = z^2 (z - z_1) (z - \frac{1}{z_1})$

(b)  $H(z) = \frac{z + a}{z - a}$

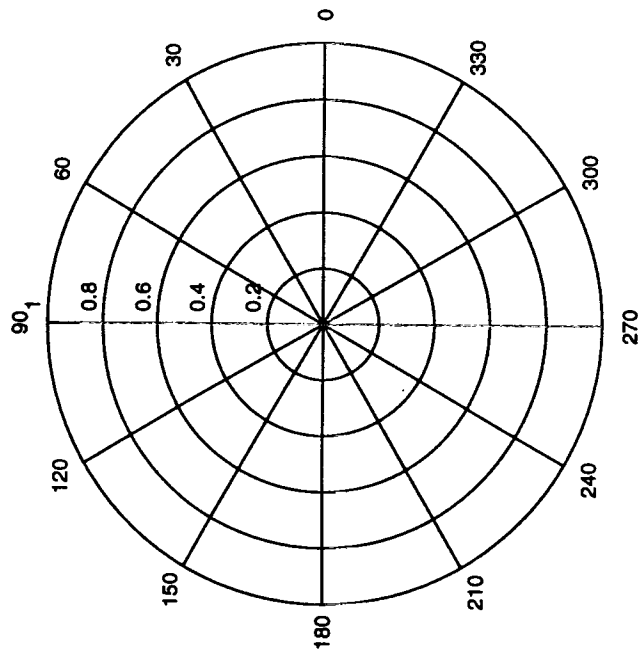
(c)  $H(z) = \frac{z^2 - a^2}{z(z + a)}$

- (2) 10. Is the system described by the system function below a minimum phase, mixed phase or maximum phase system.

$$H(z) = \frac{(z-7)(z+3)}{(z-.5)(z+.2)} \quad \mu / \text{h}$$

- (2) 11. What is the 3dB bandwidth of the low-pass filter described by.

$$H(z) = \frac{(z - e^{j\pi/2})(z - e^{-j\pi/2})}{(z - .8)^2}$$



\*\*\* The End \*\*\*



Time: 3 hours.  
Instructor: Prof. J.E. Salt  
Note: Open Book

April 26, 1994

Marks

- (15) 1. Simplify the following expressions to the extent possible.

(a)  $\sum_{n=0}^N \cos\left(\frac{2\pi n}{M}\right) \cos\left(\frac{2\pi n}{N} + \theta\right)$  where  $N, M$  are positive integers

(b)  $\sum_{n=0}^{\infty} (0.9 + j0.6)^n$

(c)  $\sum_{n=0}^{\infty} (3 + j3)^{-n}$

- (15) 2. Find the mathematical continuous time function or discrete time series, whatever the case may be, if their Fourier transforms are

(a)  $X(\omega) = e^{-\omega} u(\omega)$

(b)  $X(\omega) = 1 + \cos \omega$

(c)  $X(\omega) = \begin{cases} e^{-|\omega|} & ; |\omega| \leq \pi \\ 0 & ; \text{otherwise} \end{cases}$

Note: The argument  $\omega$  is used here in a general sense, i.e. it is also used for  $\Omega$  in which case it has units radians/sec.

- (15) 3. Find the Fourier Transforms or Fourier series coefficients, whatever the case may be.

(a)  $x(n) = \delta(n) + 7\delta(n-3) + \delta(n-6)$

(b)  $y(n) = \sum_{m=-\infty}^{\infty} x(n+9m)$ ; where  $x(n)$  is given in (a) above

(c)  $x(t) = \begin{cases} e^t & 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$

- (10) 4. (a) Is it possible for two filters with different pole/zero arrangements to have identical magnitude responses? Explain if it is or is not possible. If it is possible then give an example.

.../2

April 1994

Marks

- (b) Is it possible for two filters with different pole zero arrangements to have identical phase responses? Explain and give an example if such a filter is possible.
- (c) Is it possible to have filters that simultaneously satisfy a) and b)? Explain and give an example if such a filter is possible.

- (15) 5. (a) The system function of a filter is given by  $H(z) = 3 + z^{-1}$ . Find the output  $y(n)$  for input  $x(n)$ , where  $x(n)$  is given by

$x(n) = \cos\left(\frac{\pi n}{4} + 0.6\right) + 2$

$-\infty < n < \infty$

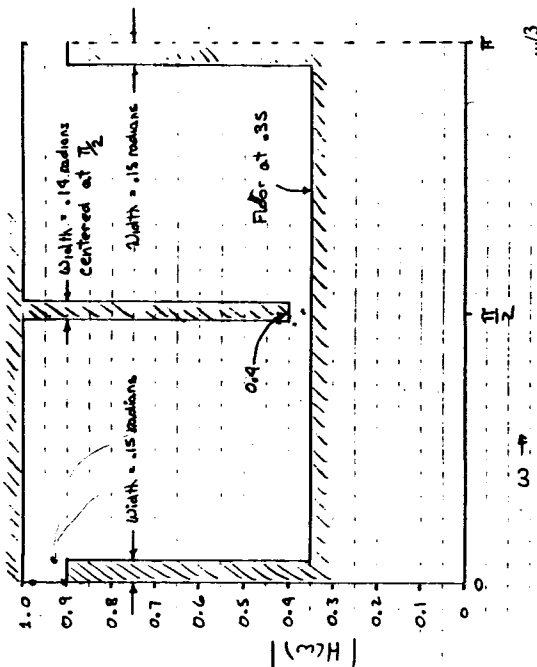
- (b) Consider the discrete time system with frequency response  $H(\omega) = 1 + e^{-j\omega}$ . Are the following three functions eigenfunctions of the system, and if so, what are the eigenvalues?

i)  $e^{j5n}$

ii)  $\cos\left(\frac{2\pi}{7}n\right)$

iii)  $\sin\left(\frac{3\pi}{28}n\right)$

- (15) 6. (a) Design a filter to the template below.



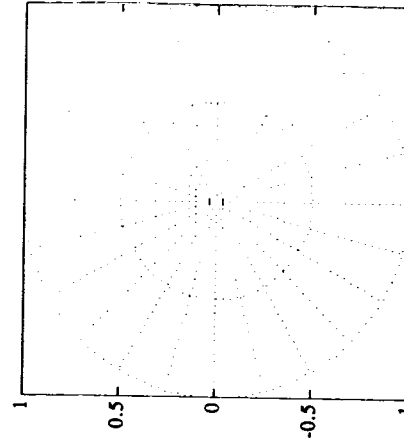
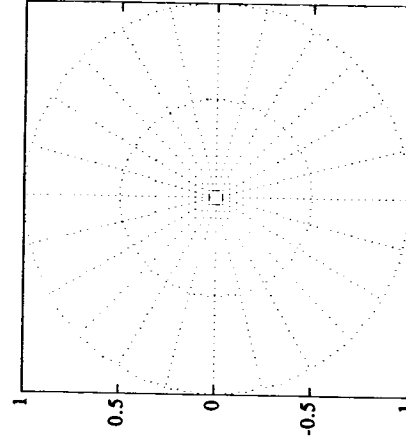
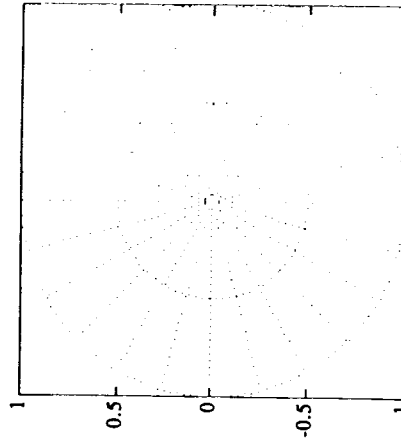
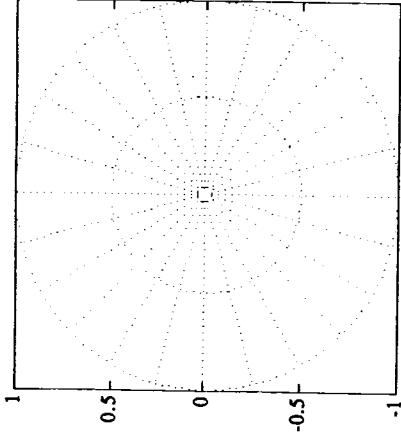
.../3

April 1994

WORKSHEET

Mark's

- (15) (b) Design an implementable filter with a bandwidth of 2Hz and a notch at 60 Hz (i.e. the 60Hz response should be zero). The sampling rate is 6000 samples per second (i.e. after normalization the 60 Hz interference is at frequency  $\frac{60}{6000} = 100$  Hz or  $\frac{2\pi}{100}$  radians). Be sure to clearly specify the location of the poles and zeros of your filter.



\*\*\* The End \*\*\*

(Worksheet attached)

EE 485: Communication/Transmission  
FINAL EXAMINATION, 9:00AM, April 29, 2002  
Time: 3 hours, closed book.

Examiner: Ha H. Nguyen

Permitted Materials: Calculator

Note: There are 5 questions. All questions are of equal value (with part marks indicated) but not necessarily of equal difficulty. Full marks shall only be given to solutions that are properly explained and justified.

1. (Ternary Modulation) Three equally probable messages  $m_1, m_2, m_3$  are to be transmitted over an AWGN channel with a two-sided PSD of  $N_0/2$ . The three signals used for transmission are:

$$s_1(t) = \begin{cases} 1, & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$s_2(t) = -s_3(t) = \begin{cases} 1, & 0 \leq t \leq T/2 \\ -1, & T/2 \leq t \leq T \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

- 2 marks (a) Sketch the three signals  $s_1(t)$ ,  $s_2(t)$  and  $s_3(t)$ .  
3 marks (b) What is the dimensionality of this signal set? Find one basis set for the signal space. Draw the signal constellation.  
2 marks (c) Draw the decision boundary and label the decision regions for the optimal receiver that minimizes the message error probability.  
1 mark (d) Which of the three signals is most susceptible to errors and why?  
2 marks (e) Compute the error probability given that the signal identified in (d) was transmitted.

2. (AM/ Alternate-Mark-Invert) is a binary line coding scheme. The output signal is determined from the source's bit stream as follows:

- If the bit to be transmitted is a 0, then the signal is 0 volts over the bit period of  $T_b$  seconds.
- If the bit to be transmitted is a 1, then the signal is either  $+V$  volts or  $-V$  volts over the bit period of  $T_b$  seconds. It is  $+V$  volts if previously a  $-V$  volts was used to represent bit 1,  $-V$  volts if previously a  $+V$  volts was used to represent bit 1. Hence the name and mnemonic for the modulation.

Now for the questions.

- 2 marks (a) Draw the three waveforms and a signal space representation of the above modulation.

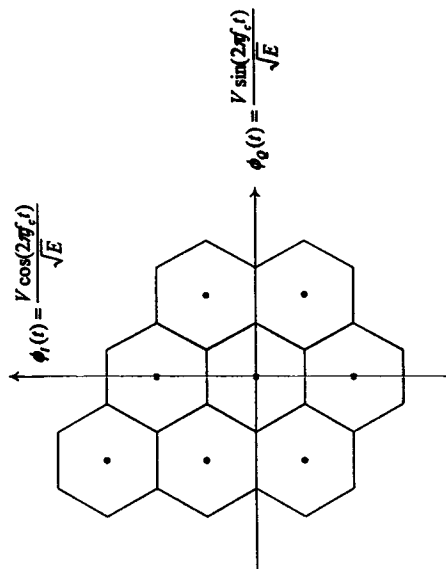
(b) Generally, the signal transmitted in any bit period depends on what happened previously. Thus there is memory and therefore a state diagram and a trellis. Draw a state diagram. As a hint, there are two states. Also a state is defined as what do you need to know from the past which together with present input (bit 1 or bit 0) enables you to determine the output ( $+V$ ,  $0$ ,  $-V$  volts). Label the transitions between the states with the input bit and the output signal.

- 2 marks (c) Now draw the trellis corresponding to the above state diagram. Start at  $t = 0$  and assume that before  $t = 0$  the voltage level corresponding to a 1 is  $+V$  volts.

- 2 marks (d) Assume that the source bits are equally likely and that  $V^2 T_b = 1$  joule. Using the signal space diagram of (a) and trellis of (c) sequence demodulate the following set of outputs from a matched filter for the first 3 bit intervals:

$$r^{(1)} = 0.4; r^{(2)} = -0.8; r^{(3)} = 0.2 \quad (\text{volts}). \quad (3)$$

3. (QAM) You are asked to design a modulation scheme for a communication system, and to conserve bandwidth it has been decided to use "QAM" modulation with an 8-point signal constellation. Unhappy with 8-ary PSK and 8-QAM because you feel that they do not use the available energy very efficiently, you decide to attempt a different signal constellation. Inspired by a tile design you notice in the local shopping mall, you propose the following signal constellation:



Assume each hexagon side is of length  $\Delta$ . Determine:



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2. The data of the sample power system shown in Figure 2 are given in Tables 1 and 2. Using Gauss-Seidel iterative algorithm, perform 2 iterations and check the convergence after each iteration. Use a voltage magnitude tolerance of 0.001, an acceleration factor of 1.6 and 100 MVA base.

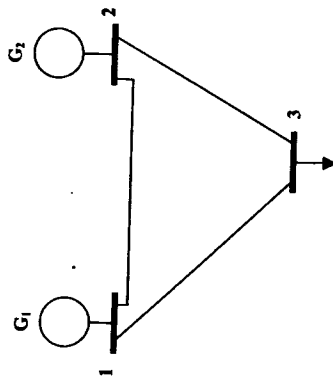


Fig. 2

Table 1: Impedances of the sample power system in p.u. on a 100 MVA base

Bus Code: p - q	Impedance $Z_{pq}$	Line charging $0.5Y_{pq}$
1-2	$0.04 + j0.16$	$j0.15$
1-3	$0.02 + j0.08$	$j0.07$
2-3	$0.05 + j0.12$	$j0.08$

Table 2: Scheduled generation and loads and magnitudes of bus voltages for the sample power system.

Bus code p	Bus voltage	Generation		Load	
		MW	MVAR	MW	MVAR
1	1.04	?	?	0	0
2	1.02	40	?	0	0
3	?	0	0	100	40

(12 Marks)

3. In the system shown in Figure 3, a three-phase fault occurred on one of the transmission lines just after the circuit breaker. Find the following:
- The critical clearing angle in degrees.
  - The critical clearing time in seconds.
  - The generator speed at the instant of clearing in radians per second.

$$x_d = j0.4 \text{ p.u.}, \quad x_{T1} = j0.8 \text{ p.u.}, \quad x_{T2} = j0.1 \text{ p.u.}, \quad M = 7 \text{ sec}$$

- 1- Consider the power system shown in Fig. 1. Use a power base of 500 MVA and network reduction to calculate the fault current in Amperes and the line-to-line voltages at the fault point for a sustained single line-to-ground fault at bus D.

$G_1$ : 500 MVA, 13.8 kv,  $x_d'' = 0.2 \text{ p.u.}$ ,  $x_2 = 0.2 \text{ p.u.}$  and  $x_o = 0.1 \text{ p.u.}$   
 $G_2$ : 600 MVA, 26 kv,  $x_d'' = 0.15 \text{ p.u.}$ ,  $x_2 = 0.15 \text{ p.u.}$  and  $x_o = 0.1 \text{ p.u.}$   
 $G_3$ : 400 MVA, 13.8 kv,  $x_d'' = 0.2 \text{ p.u.}$ ,  $x_2 = 0.2 \text{ p.u.}$  and  $x_o = 0.1 \text{ p.u.}$   
 $T_1$ : 500 MVA, 13.8 kv/500 kv,  $x = 0.1 \text{ p.u.}$   
 $T_2$ : 600 MVA, 26 kv/500 kv,  $x = 0.1 \text{ p.u.}$   
 $T_3$ : 500 MVA, 13.8 kv/500 kv,  $x = 0.1 \text{ p.u.}$   
Line AB,  $x_1 = 50 \Omega$   
Line BC,  $x_1 = 80 \Omega$   
Line AD,  $x_1 = 80 \Omega$   
Line BD,  $x_1 = 120 \Omega$

Mid term: Solve above using bus admittance matrix. Calculate bus voltage at bus A

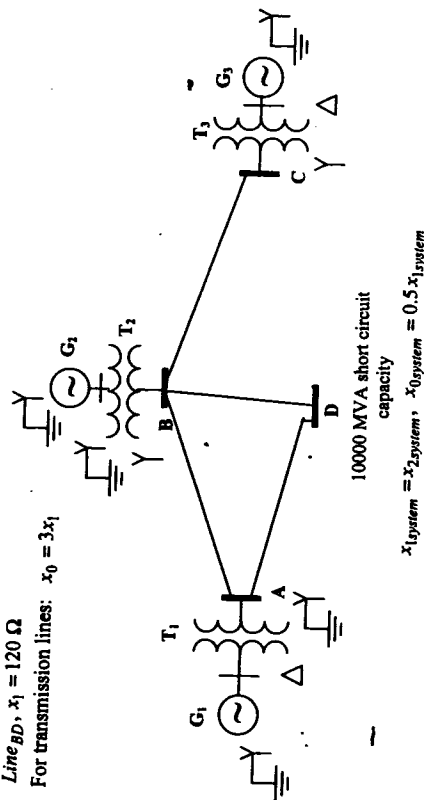


Fig. 1

(12 Marks)

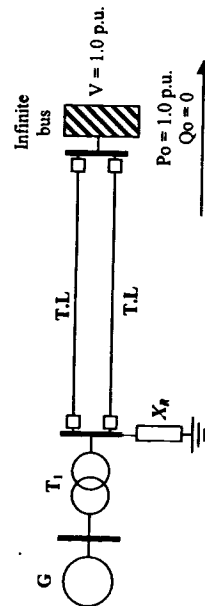


Fig. 6

(6 Marks)

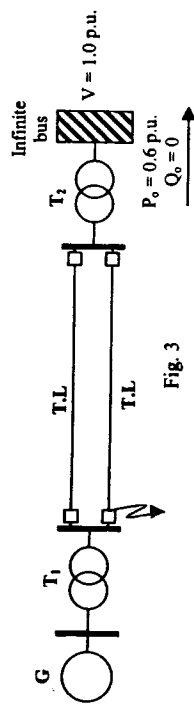


Fig. 3

(12 Marks)

4. In the system shown in Figure 4, a three-phase fault occurred on one of the transmission lines at the middle point. The switch S is opened simultaneously with circuit breakers A and B. Find the critical clearing angle.

$$x_d = j0.4 \text{ p.u.}, \quad X_C = -j0.1 \text{ p.u.}, \quad x_{T.L} = j1.0 \text{ p.u. (each of the four sections)}$$

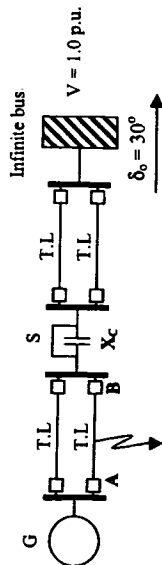


Fig. 4

(12 Marks)

5. Consider the system shown in Figure 5. Using the equal area criterion, discuss whether the transformer neutral reactance  $X_T$  improves or degrades the system transient stability.

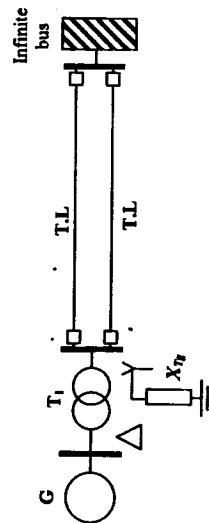


Fig. 5

(6 Marks)

6. Consider the system shown in Figure 6. Find the synchronizing power and the natural frequency of free oscillations.

$$x_d = j1.0 \text{ p.u.}, \quad x_{T.L} = j0.8 \text{ p.u.}, \quad x_{T1} = j0.1 \text{ p.u.}, \quad x_R = j0.5 \text{ p.u.}, \quad M = 7 \text{ sec}$$

Instructor: S.O. Faried  
Duration: 2 hours

October 25, 2001

1. For the system shown in Fig. 1, sketch the root locus showing all the pertinent characteristics.

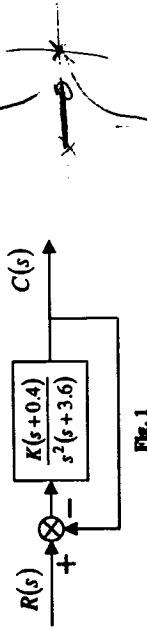


Fig. 1

2. Consider the closed-loop transfer function:

$$\frac{C(s)}{R(s)} = \frac{0.25K(s+0.435)}{s^4 + 3.456s^3 + 3.457s^2 + (0.719 + 0.25K)s + (0.0416 + 0.109K)}$$

Find the range of  $K$  that ensures that the closed-loop control system is stable.

3. Consider the control system shown in Fig. 2(a). Design a rate feedback compensation, as shown in Fig. 2(b), to reduce the settling time by a factor of 4 while continuing to operate the system with the same overshoot.

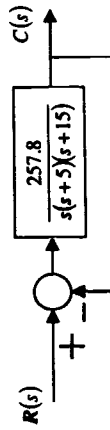


Fig. 2(a)

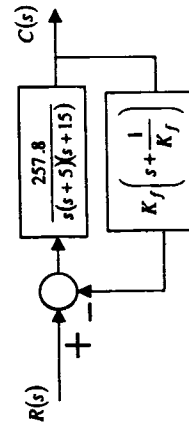


Fig. 2(b)

4. Fig. 3 shows open-loop poles and zeros. There are two possibilities for the sketch of the root locus. Sketch each of the two possibilities. Be aware that only one can be the *real* locus for specific open-loop pole and zero values.

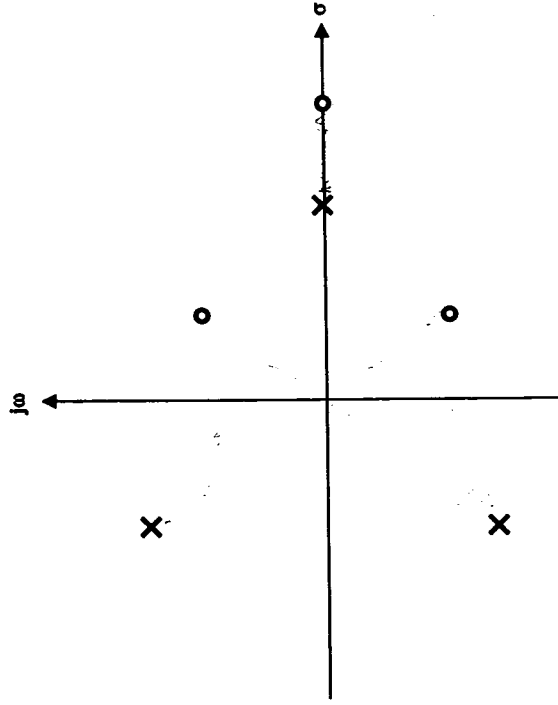
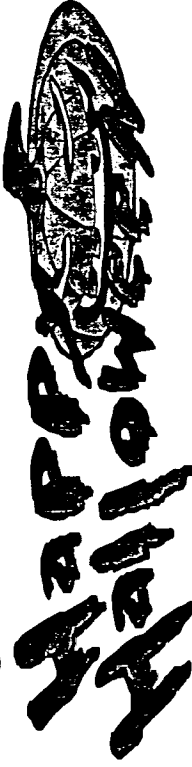
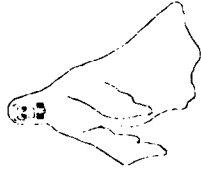


Fig. 3



Instructor: Sherif O. Faried  
Three formula sheets are allowed

Duration: 3 hours  
December 8, 2001

- Consider the system shown in Fig. 1. It is desired to design a PID controller  $G_c(s)$  such that the dominant closed-loop poles are located at  $s = -1 \pm j\sqrt{3}$ . For the PID controller, choose  $a = 1$  and then determine the values of  $K$  and  $b$ . Sketch the root-locus diagram for the designed system.

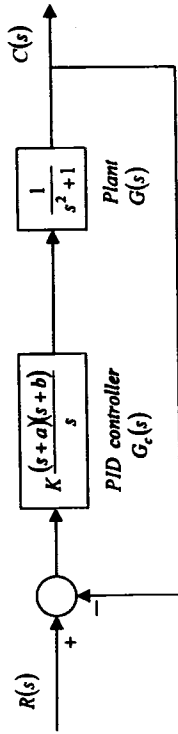


Fig. 1.

An equivalent block diagram of the pitch attitude axis of a Remotely Piloted Aircraft (RPA) is illustrated in Fig. 2. The transportation lag  $T_1$  represents the delay caused by the man-in-the-loop at the ground station and the time it takes to transmit the signal from the ground station to the RPA. The transportation lag  $T_2$  represents the time it takes for the return signal to be received by the ground station from the RPA. Assume that  $T_1 = 0.3$  sec and that  $T_2 = 0.05$  sec.

- Determine analytically the gain crossover frequency needed to achieve a phase margin of  $50^\circ$ .
- Determine the value of  $K$  needed to obtain the gain crossover frequency obtained in part (a).

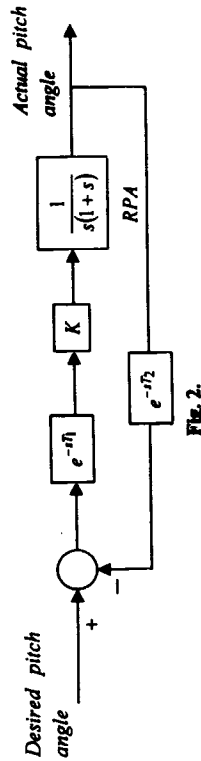


Fig. 2.

- Using the Routh's stability criterion, draw a conclusion about the stability of the closed-loop system shown in Fig. 3.

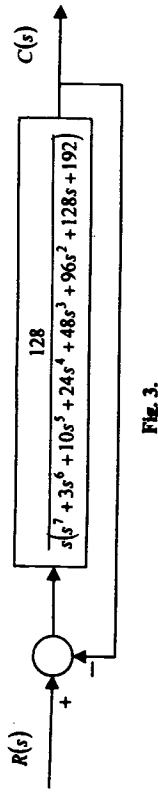


Fig. 3.

- The block diagram of a positioning system is shown in Fig. 4.  
(a) Without any compensation, ( $G_c(s) = 1$ ), draw the root locus of the uncompensated system showing all the pertinent characteristics.  
(b) Determine the value of  $K$  such that the damping ratio  $\xi$  of the closed-loop complex poles is 0.707.

- It is desired to increase the static velocity error constant  $K_v$  to about  $3.75 \text{ sec}^{-1}$  without appreciably changing the location of the dominant closed-loop poles. Using the root-locus method, determine the compensator  $G_c(s)$  which can achieve this. Find approximately its angle contribution near the dominant closed-loop poles.

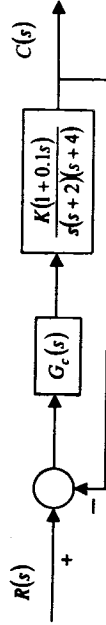


Fig. 4.

For the system shown in Fig. 5, sketch the root locus showing all the pertinent characteristics.

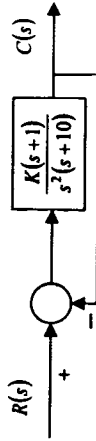
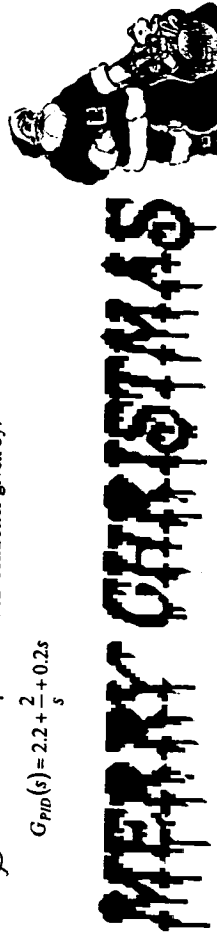


Fig. 5.

- Fig. 6 shows the Bode diagram of  $G(s)H(s)$  of a negative feedback control system. Determine the maximum overshoot for a unit-step response.
- Sketch the Bode plots of a PID controller given by:

$$G_{PID}(s) = 2.2 + \frac{2}{s} + 0.2s$$





Instructor: Sherif O. Faried  
A one formula sheet is allowed

Duration: 90 minutes  
October 23, 2000

- For the system of Figure 1, find the values of  $K_1$  and  $K_2$  to yield a peak time of 1 second and a settling time (2% criterion) of 2 seconds for the closed-loop system's step response.

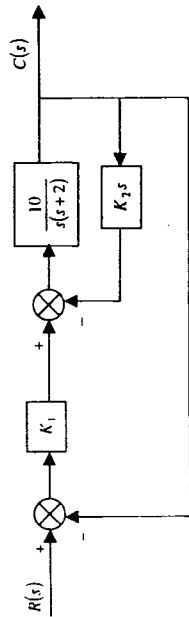


Figure 1.

- Use the Routh-Hurwitz criterion to find the range of  $K$  for which the system of Figure 2 is stable.

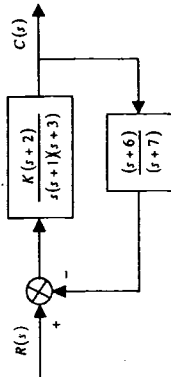


Figure 2.

- For the system shown in Figure 3, sketch the root locus showing all the pertinent characteristics and find the range of  $K$  within the system is stable.

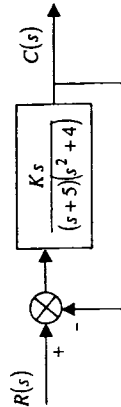


Figure 3.



Instructor: Sherif O. Faried  
A one formula sheet is allowed

Duration: 3 hours  
December 2000

- Find the following for the system shown in Figure 1:

- The transfer function  $T(s) = \frac{C(s)}{R(s)}$ .
- The damping ratio, percent overshoot, settling time (2% criterion), peak time and rise time.

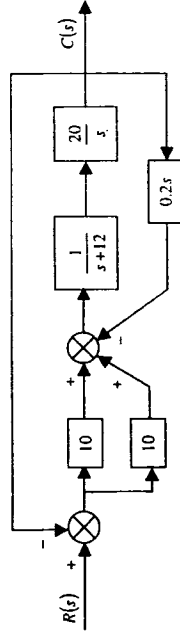


Figure 1

(10 Marks)

- Consider the control system shown in Figure 2.

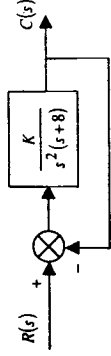


Figure 2

- Sketch the root locus and indicate **all** pertinent characteristics of the locus. Discuss the effect of the gain  $K$  on the system stability.
- If  $K = 4$ , design a compensator such that the dominant closed loop poles are located at  $s = -1 \pm j\sqrt{3}$ . Your design should lead to the maximum possible value of the static velocity error constant. Determine this maximum value.
- Sketch the root locus of the new compensated system and indicate **all** pertinent characteristics of the locus.

(16 Marks)

- Consider a unity negative feedback system with

$$G(s) = \frac{K}{(s+3)(s+5)}$$

Show that the system cannot operate with a settling time (2% criterion) of 0.667 second and a percent overshoot of 1.5% with a simple gain adjustment.

(8 Marks)

4. For the system shown in Figure 3:

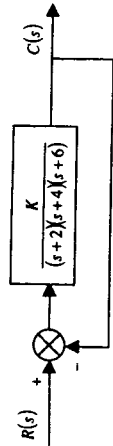


Figure 3

- Sketch the Bode plots of the open-loop transfer function.
- Sketch the Nyquist diagram.
- With the help of the Nyquist diagram, find analytically the range of gain  $K$ , for stability. (a zero mark will be given if you use Routh's stability criterion).
- Find the gain margin if  $K = 100$ .

(10 Marks)

5. Consider a system having the open-loop transfer function

$$GH(s) = \frac{1}{s^4(s+p)}, \quad p > 0.$$

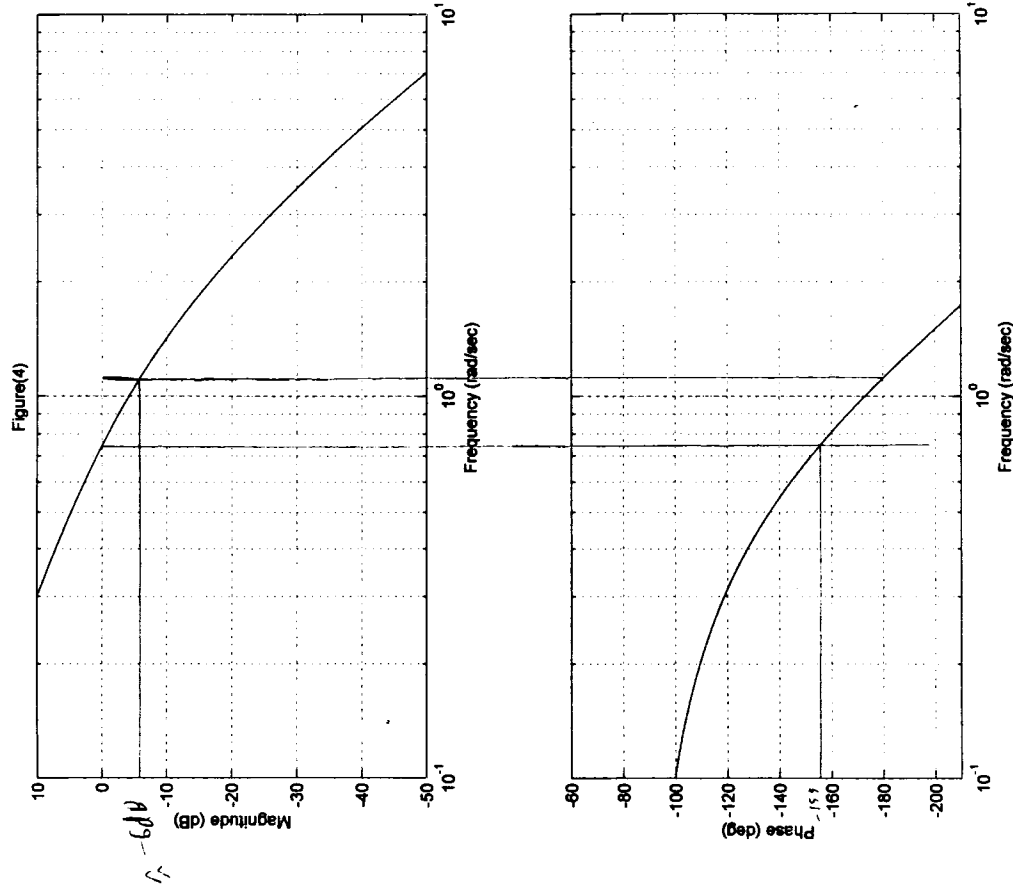
- Sketch the Bode plots of the open-loop transfer function.
- Sketch the Nyquist diagram.
- Determine  $N$ ,  $P$  and  $Z$  and discuss the stability of the system.

(8 Marks)

6. The Bode plots for a plant  $G(s)$ , used in a unity negative feedback system are shown in Figure 4.

- Find the gain margin and the phase margin.

(8 Marks)



- ✓ The transient response of the control system in Fig. 1 is to be analyzed.

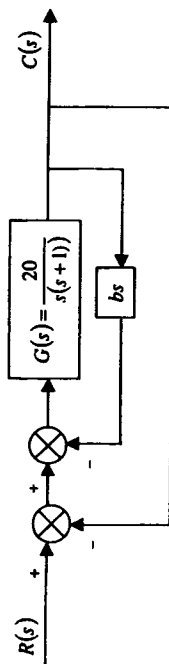


Fig. 1.

- (a) Determine the rate feedback constant  $b$  so that this control system has a damping ratio of 0.7.

- (b) Determine the rise time,  $t_r$ , the time to peak,  $t_p$ , and the settling time,  $t_s$  (2% criterion).

2. The pitch attitude control system for a booster rocket containing attitude and rate gyros is shown in Fig. 2. Sketch the root locus and determine the maximum value of  $K$  that would permit stable operation.

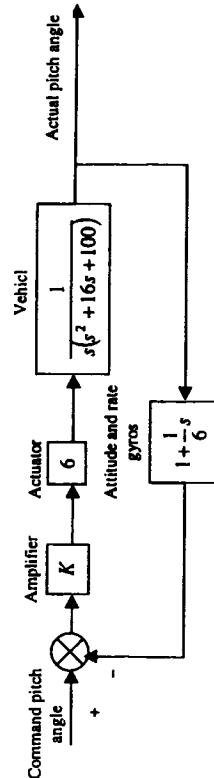


Fig. 2.

3. A unity-feedback control system has the following forward transfer function:

$$G(s) = \frac{K(s+a)}{s^2(s+2)}$$

Determine the values of  $a$  so that the root locus will have zero breakaway point, not including the one at  $s=0$ .

4. It is desired to analyze the performance of a unity-feedback second-order system whose forward transfer function represents a process  $G_p(s)$ , given by:

$$G_p(s) = \frac{500}{s(s+10)}$$

- (a) Determine the gain crossover frequency,  $\omega_c$ , analytically.  
(b) Determine the phase margin and gain margin of this control system analytically.  
(c) A phase-lead network compensation,  $G_c(s)$ , given by:

$$G_c(s) = \frac{(1+aTs)}{(1+Ts)}$$

is to be added in series with the process' transfer function,  $G_p(s)$ . Determine the values of  $a$  and  $T$  in order that the zero factor of  $G_c(s)$  cancels the pole of  $G_p(s)$  at  $s = -10$ , and the damping ratio of the control system is unity.

- (d) Determine analytically the gain crossover frequency,  $\omega_c$ , for the compensated system.  
(e) Determine the phase margin and gain margin of this compensated control system analytically.

5. Determine the values of  $K$ ,  $T_1$ , and  $T_2$  of the system shown in Fig. 3 so that the dominant closed-loop poles have the damping ratio  $\xi = 0.5$  and the undamped natural frequency  $\omega_n = 3 \text{ rad/sec}$ . Moreover, it is required to have the maximum possible value of the static velocity error constant. Determine this maximum value.

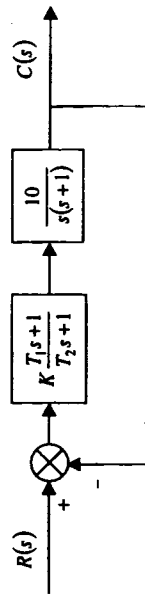
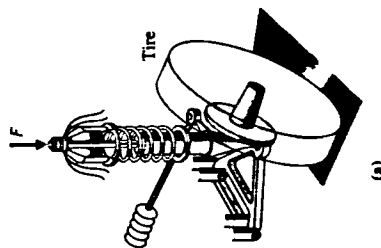


Fig. 3.

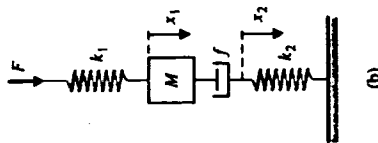
$$G_c(s) =$$

1. A load added to a truck results in a force  $F$  on the support spring and the tire flexes as shown in Fig. P2.47(a). The model for the tire movement is shown in Fig. P2.47(b).

- a) Determine the differential equation relating the displacement of the mass  $M$  and the applied force  $F$ .
- b) Determine the transfer function  $X_1(s)/F(s)$ .



(a)



(b)

FIGURE P2.47 Truck support model.

2. An ideal set of gears is connected to a solid cylinder load as shown in Fig. P2.45. The inertia of the motor shaft and gear  $G_1$  is  $J_m$ . Determine (a) the inertia of the load  $J_L$  and (b) the torque  $T$  at the motor shaft. Assume the friction at the load is  $f_L$  and the friction at the motor shaft is  $f_m$ . Also assume the density of the load disk is  $\rho$ .

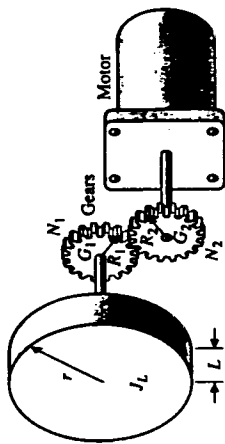


FIGURE P2.45 Motor, gears, and load.

## EE 410.3 Controls I

## Quiz # 2

October 1997

Do Both Questions:

1. A control system has the structure shown in Fig. 1.
- Determine the closed loop transfer function  $C(s)/R(s)$  using the method of block diagram manipulation.
  - Select gains  $K_1$  and  $K_2$  so that the closed loop response to a step input is critically damped with two equal roots at  $s = -10$ .
2. The circuit shown in Fig. 2 is called a lead-lag filter.
- Find the transfer function  $V_2(s)/V_1(s)$  using the signal flow graph method and Mason's rules. (Draw the flow graph and indicate how you find the transfer function.)
  - Confirm the result of part a) using any other method to find the same transfer function.

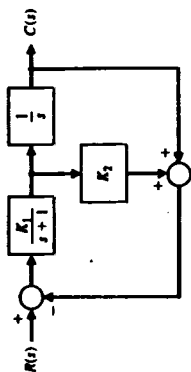


Figure 1.

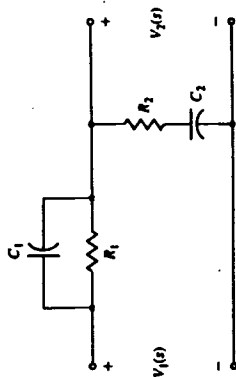
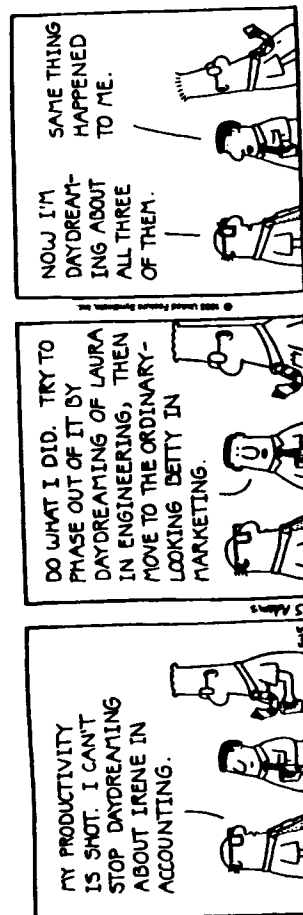


Figure 2.



November 1996

Notes:  
Instructor: H. Wood  
Time: 85 minutes  
Notes: 2 or 3 pages  
Marks: As indicated; Do all 3 questions.

1. The objective of this question is to design a controller for the system illustrated in Figure 1. The controller is required to have a DC gain of  $K$ , and must have a single pole at a location  $b$  on the left hand side of the  $s$ -plane. To solve for the two unknown factors in the controller, two design criteria are given. The steady state error in response to a unit step input is 20%, and the system must be stable.

- What is the expression for the transfer function of the controller itself?
- Show that the DC gain of your controller is in fact  $K$ .
- Find the closed loop transfer function  $T(s)$ .
- What is the expression for the steady state error of the closed loop system in response to a step input?
- Use the steady state error limit of 0.2 to evaluate one of the controller unknowns (it should be clear from the expression for the s.s.e. which one).
- Use the stability criterion to find the range of acceptable values for the second controller unknown.

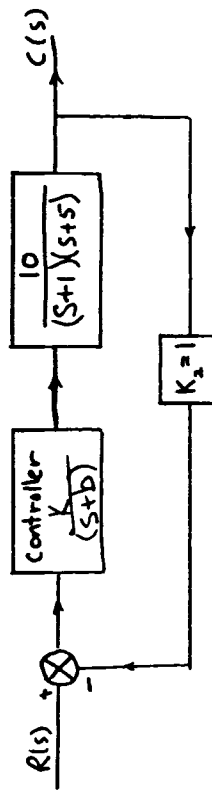


Figure 1

2. A controller with a single pole at  $s = -100$  and a gain factor of  $K$  is used to provide an input signal to a plant. Unity gain negative feedback is established by comparing the output signal  $C(s)$  with the reference input signal  $R(s)$ . When a step input is applied to the OPEN loop system, the response is as shown in Figure 2.

- Assume the response is approximately second order. What are the natural frequency and the damping ratio for the plant?
- What is the value of the gain factor for the controller?  
(Hint: Use the Final Value Theorem and the illustrated response)
- Show all of the root locations in the  $s$ -plane for the open loop system.
- Is the assumption made in part a) justified? Why or why not?
- Now connect the feedback and determine the closed loop transfer function.
- Again assuming the system is approximately second order, what is the natural frequency of the closed loop system? How does it compare with the open loop system frequency? Do you expect this result for the comparison? Why?

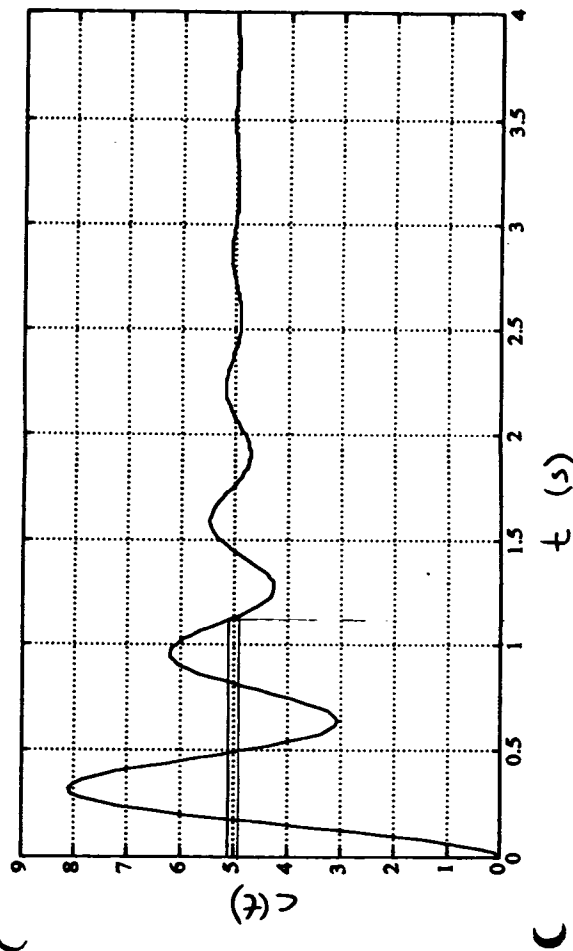
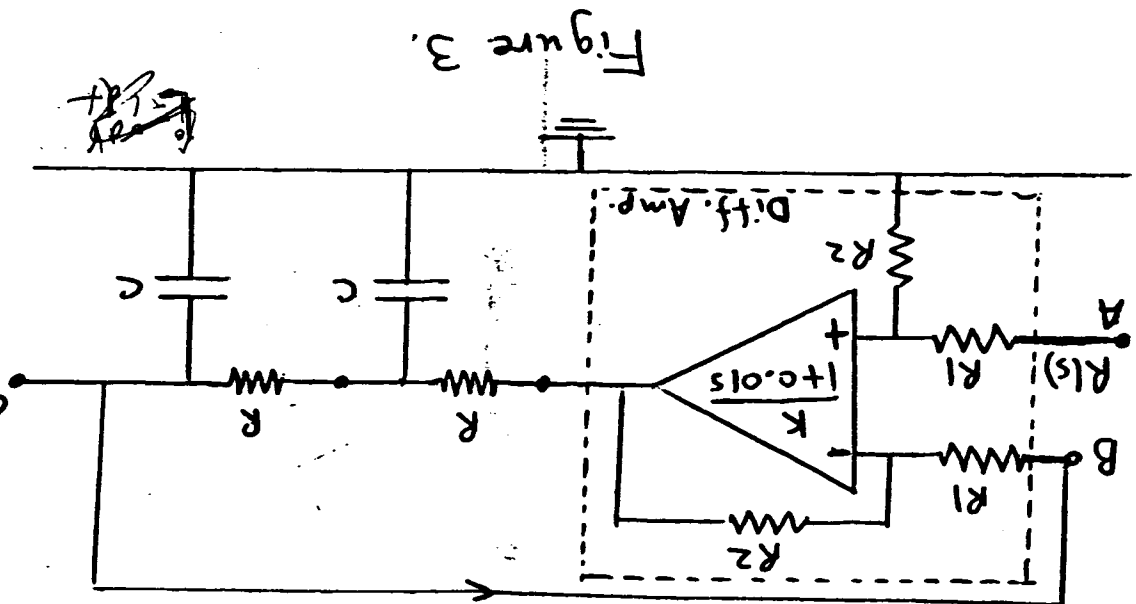


Figure 2.

3. The operational amplifier circuit in Figure 3 consists of a differential amplifier followed by two separate but equivalent filter units. The differential amplifier, in the configuration shown, multiplies the voltage difference at its input terminals A and B by the DC gain factor  $K$ . The operational amplifier has an effective time constant of 0.01 seconds. The filter resistor values are each 10 kOhms and the capacitor values are 2.0 microFarads.

- Draw the block diagram of the closed loop system.
- What is the maximum value of the DC gain of the amplifier for stability?
- At the maximum gain, at what frequency will the circuit oscillate?



$$V = ? \left( R + \frac{1}{sC} \right)$$

$$V \cdot \frac{sC}{R + \frac{1}{sC}} = \frac{sRC + 1}{sRC + 1}$$

$$V = \frac{sRC + 1}{sRC + 1}$$

$$\frac{1}{sC}$$

$$V = \frac{1}{sC}$$

$$V = L \frac{dv}{dt}$$

October 27, 1999

Instructor: S.O. Faried

Time: 90 minutes

Note: One formula sheet is allowed

1. Consider the system shown in Figure 1. Determine the range of values of  $K$  for which the system is stable.

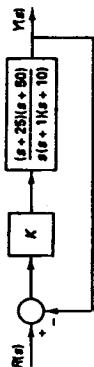


Figure 1

2. Sketch the root locus for a unity-feedback control system whose forward transfer function is given by

$$G(s) = \frac{K}{(s+2)^4}$$

At what value of  $K$  does the system become unstable, and where does the root locus intersect the  $j\omega$  axis when this occurs?

3. Sketch the root locus for a unity-feedback control system whose forward transfer function is given by

$$G(s) = \frac{K(s+2)}{s^2(s+18)}$$

- (i) Determine the location of the roots when all three roots are all real and equal.
- (ii) Find the gain when all the roots are real and equal.

The End

Your task today is to design a control system for a new electric car. The car, with a total mass of 800 kg, is battery operated and all of the controls are electrical or electronic. The car is driven by an electric motor whose output torque is proportional to the current through the motor. The motor is connected to the wheels through a gear reduction of 5:1 (motor shaft turns 5 times as fast as the axle), and the wheel diameter is 36 cm. The electric motor can be modelled as a resistance  $R$  in series with an inductance  $L$ . The vehicle experiences air friction and rolling friction, all combined in one term that is directly proportional to speed.

To control the speed of the vehicle, a power control unit outputs a voltage to the motor that is directly proportional to the angular position of a manually operated dial on the control unit.

**Note:** This question has many parts; each part is really a continuation of the same problem, but, it is not necessary to get each part correct to proceed to the next part. Each succeeding part starts from an assumed solution to the previous part that is given to you. This solution is not necessarily the actual solution to the previous part, but gives everyone the same starting point for the next part. Even if you think you have the correct solution to a part, do not use your solution for the next part, but instead use the one given to you.

1. Assume the vehicle is at rest at time  $t=0$  and the dial is set to 0.

Draw a sketch of the system to help you visualize what is going on.

- a) Develop the differential equation that relates the torque produced by the motor to the position of the vehicle. (ignore rotational inertias)
- b) Develop the differential equation that relates the angular position of the 'speed dial' on the controller to the motor torque.
- c) Put these equations together to give an equation relating the dial setting to the vehicle position.

2. Assume that the solution to 1 c) is as follows: ( $d$  is dial position,  $x$  is vehicle position).

$$d(t) = A x'''(t) + B x''(t) + C x'(t) \quad \text{where } x' \text{ represents } dx/dt.$$

- a) Determine the Laplace transform expression relating the variables  $x$  and  $d$ , assuming zero initial state for the system.
- b) Determine the Laplace transform expression relating the vehicle speed  $v$  to the dial setting, now assuming that the vehicle is moving at uniform speed  $v(0)$  at time  $t=0$ .

**Note:** Use degrees throughout; do not change to radians.

University of Saskatchewan  
College of Engineering  
EE 444.3: Electrical Machines II  
Midterm Examination

Instructor: Dr. N. Kar

Time: 1 hour & 20 min.

Note: One sheet of handwritten formulas permitted

October 29, 2002

Marks

- 20 1. (a) Calculate the force produced on the moving part of the shown unipivot relay mechanism (Fig. 1) where the motion may be assumed to be linear. The coil has 1000 turns and the DC current flowing in it is 1.0 A. Neglect fringing and leakage flux, and assume that all the energy is stored in the air-gap.

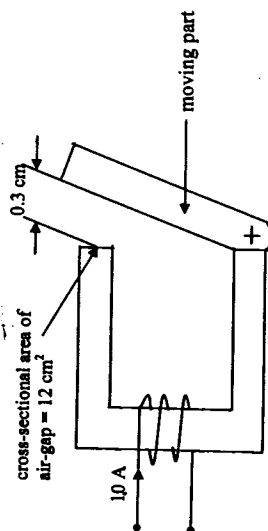


Fig. 1. Relay mechanism.

- (b) If the following factors:

- (i) the leakage flux
- (ii) the fringing effect
- (iii) the iron path of the magnetic path

are not neglected, describe using literature the effect of these factors on the value of the force calculated in (a).

- (c) Answer whether the following statements are true or false.

- (i) If the magnetization curve of an electromagnetic device is nonlinear, the energy stored in the magnetic field is smaller than the coenergy.
- (ii) The synchronous reactance of a synchronous generator is larger than its leakage reactance.
- (iii) A synchronous generator operating at lagging PF (power factor) is underexcited.

- 30 2. A 480 V, 60 Hz,  $\Delta$ -connected, 4-pole synchronous generator has the open-circuit characteristic shown in Fig. 2. This generator has a synchronous reactance of  $0.11 \Omega$  and an armature resistance of  $0.016 \Omega$ . At full-load, the machine supplies 1200 A at 0.8 PF lagging. Under full-load conditions, the friction and windage losses are 40 kW, and the core losses are 30 kW. Ignore any field circuit losses.

- (a) What is the speed of rotation of this generator? ✓
- (b) How much field current must be supplied to the generator to make the terminal voltage 480 V at no load? ✓
- (c) If the generator is now connected to a load and the load draws 1200 A at 0.8 PF lagging, how much field current will be required to keep the terminal voltage equal to 480 V. Draw the phasor diagram. ✓
- (d) How much power is the generator now supplying? How much power is supplied to the generator by the prime-mover? What is the machine's overall efficiency? ✓
- (e) If the generator's load were suddenly disconnected from the line, what would happen to its terminal voltage? ✓
- (f) Finally, suppose the generator is connected to a load drawing 1200 A at 0.8 PF leading. Draw the phasor diagram. How much field current would be required to keep the terminal voltage at 480 V? ✓

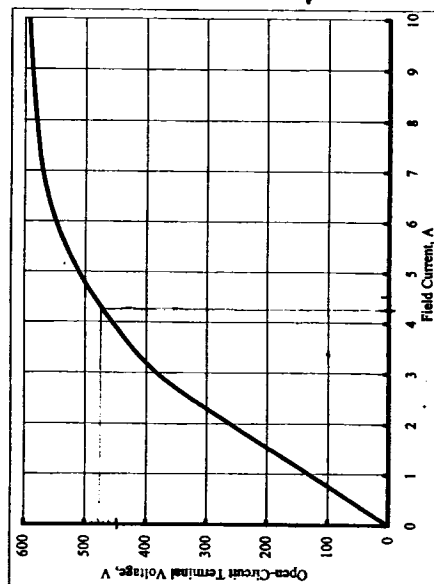


Fig. 2. Open-circuit characteristic of the generator in Question 2.

— The End —



- 20 3. (a) What are the advantages and disadvantages of brushless dc motors compared to ordinary brush dc motors?
- (b) A 460-V, 25-hp, 60-Hz, 4-pole, Y-connected, wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641 \, \Omega \quad R_2 = 0.332 \, \Omega \quad X_M = 26.3 \, \Omega$$

$$X_1 = 1.106 \, \Omega \quad X_2 = 0.464 \, \Omega$$

- i) What is the maximum torque of this motor? At what speed and slip does it occur?
- ii) What is the starting torque of this motor?
- iii) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

- 15 4. (a) Neglecting the stator resistance, show that the active power output of a cylindrical-rotor synchronous generator connected to an infinite bus is given by

$$P = \frac{E_f V_t}{X_s} \sin \delta$$

- (b) Describe the effect of the excitation on the synchronous generator performance using phasor diagram when the generator real power output, frequency and terminal voltage are constant.

- 25 5. A 2000-hp, 1.0-power factor, 3-phase, Y-connected, 2300-V, 30-pole, 60-Hz synchronous motor has a synchronous reactance of  $1.95 \, \Omega/\text{phase}$ . For this problem all losses may be neglected.
- (a) Compute the maximum torque which this motor can deliver if it is supplied with power from a constant frequency source, commonly called an *infinite bus*, and if its field excitation is constant at the value which would result in 1.0 power factor at rated load.
- (b) Instead of the infinite bus of part (a) suppose that the motor is supplied with power from a 3-phase, Y-connected, 2300-V, 1750-kVA, 2-pole, 3600-r/min turbine generator whose excitations of the generator and motor are adjusted so that the motor runs at 1.0 power factor and the mechanical load on the synchronous motor is gradually increased. Compute the maximum motor torque under these conditions and the terminal voltage when the motor is delivering its maximum torque.

— THE END —

Instructor: Dr. N. Kar  
Time: 3 hours  
Note: Two sheets of handwritten formulas permitted.

December 20, 2002

Marks

- 20 1. The dimensions of electromagnet shown in Fig. 1 are in centimetre (cm) and the depth of the core and the armature is 5 cm. The coil has 1000 turns. Assuming that the permeability of the magnetic material is very large relative to air ( $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ ) and neglecting the leakage flux and the fringing of flux at the air-gaps:
- (a) Determine the required D.C. current in the coil to provide a total pull on the armature (supported by springs) of 50 N at an air-gap length of  $l = 0.8 \text{ cm}$ .
- (b) If the coil is excited from an A.C. supply, what will be the current in this case?

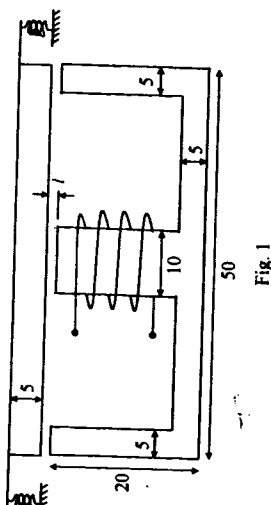


Fig. 1

- 20 2. Fig. 2 depicts a simple, single-phase, 4-pole reluctance motor. A current of 1 A at 60 Hz is passed through its stator winding. Assuming a sinusoidal variation of inductance of this winding in terms of  $\theta$ , between the maximum value of 0.4 H and a minimum value of 0.1 H:
- (a) Derive an expression as a function of time for the torque produced by this motor.
- (b) Determine the value of the speed at which this motor will develop an average torque. What will be the maximum value of this average torque at this speed?
- (c) What are the frequencies of the time varying components of the produced torque? What are the amplitudes of these components?

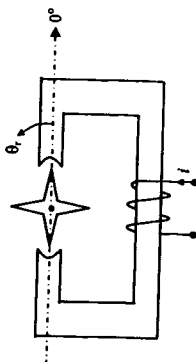


Fig. 2

Instructor: S.O. Faried  
Duration: 80 minutes

October 30, 2001

1. A 0.25 hp, 110-V, 60-Hz, four-pole capacitor-start single-phase induction motor has the following parameters and losses:

$$R_1 = 2 \Omega \quad X_{11} = 2.8 \Omega \quad X'_{12} = 2 \Omega \quad R'_2 = 4 \Omega \quad X_m = 70 \Omega$$

Core loss at 110 V = 25 W; Friction and windage = 12 W

For a slip of 0.05, compute the input current, power factor, power output, speed, torque and efficiency when the motor is running at rated voltage and rated frequency with its starting winding open.  $I = 3.567 \angle -48.8^\circ$   $P_{out} = 181 \text{ W}$   $P_{mech} = 181 \text{ W}$   $\eta = 74.4\%$

2. A 3-phase, squirrel-cage induction motor has a starting torque of 1.75 p.u. and a maximum torque of 2.5 p.u. when operated from rated voltage and frequency. The full-load torque is considered as 1 p.u. of torque. Neglect stator resistance.

(a) Determine the slip at maximum torque. 0.7

(b) Determine the slip at full-load torque. 0.28

- (c) Determine the rotor current at starting in p.u. Consider the full-load rotor current as 1 p.u.  $I_r = 3.57 \text{ p.u.}$

3. A 500 hp, 3-phase, 2200-V, 60-Hz, 12-pole, Y-connected, wound rotor induction motor has the following parameters:

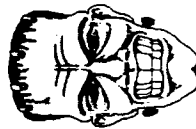
$$R_1 = 0.225 \Omega \quad R'_2 = 0.235 \Omega \quad X_{11} + X'_{12} = 1.43 \Omega \quad X_m = 31.8 \Omega$$

Use an appropriate equivalent circuit to calculate the following:

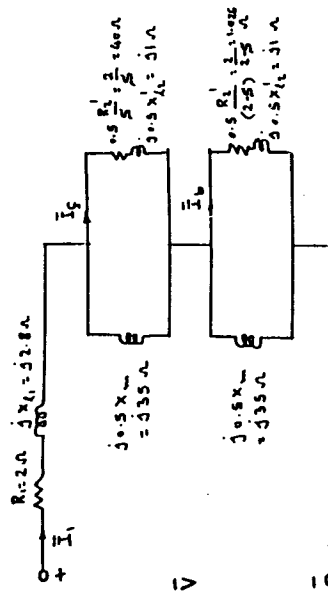
- (a) Slip at maximum torque. 0.084

- (b) Maximum torque. 1054.9 N.m

- (c) Resistance that must be added to the rotor windings (per phase) to achieve maximum torque at starting.  $R_{add} = 2.81 \Omega$



S.O. Faried's Answers  
EE453 Oct 30, 2001



$$Z_f = R_f + jX_f = \frac{(4.0 + j1)j35}{4.0 + j36} = 16.92 + j19.77$$

$$Z_b = R_b + jX_b = \frac{(1.026 + j1)j35}{1.026 + j36} = 0.97 + j31$$

$$Z_e = Z_1 + Z_f + Z_b = (2 + j3.8) + 16.92 + j19.77 + 0.97 + j31 = 19.89 + j33.57 = 30.86 \angle 59.4^\circ$$

Input Current =  $110 / 30.86 \angle 59.4^\circ = 3.57 \angle -59.4^\circ \text{ A}$

The input current is 3.57 A;  $\phi = \cos 59.4^\circ = 0.51$  lagging  
Developed Power  $P_d = (I_s^2 R_f) (1 - S) + (I_s^2 R_b) [1 - (2 - S)] = I_s^2 (R_f - R_b) (1 - S)$   
 $= 3.57^2 (16.92 - 0.97) (1 - 0.05) = 193 \text{ W}$

Shaft output power  $P_o = P_d - P_{mech} = 193 - 12 = 181 \text{ W}$

Speed:  $(1 - S) 50 = 0.95 \times 120 \times 60 / 4 = 1710 \text{ rpm}$

Torque:  $181 / 179 = 0.87 \text{ N.m}$   $\eta = 181 / (110 \times 3.57 \times 0.51) = 0.616$

(2) Neglect  $R_1$

$$T_{\text{starting}} = \frac{3 V_{th}^2 R_2'}{w_s [(R_2')^2 + (X_{eq})^2]} \quad \text{Eq. ①}$$

$$T_{\text{max}} = \frac{3 V_{th}^2}{2 w_s [X_{eq}]} \quad \text{Eq. ②}$$

$$S_{\text{max}} = \frac{R_2'}{X_{eq}} \Rightarrow R_2' = S_{\text{max}} X_{eq}$$

$$\frac{T_{\text{max}}}{T_{\text{starting}}} = \frac{\cancel{3 V_{th}^2} \cdot \cancel{w_s} [R_2'^2 + X_{eq}^2]}{2 \cancel{w_s} X_{eq} \cdot \cancel{3 V_{th}^2} R_2'}$$

$$\frac{T_{\text{max}}}{T_{\text{st}}} = \frac{1}{2 X_{eq}} \frac{[R_2'^2 + X_{eq}^2]}{R_2'}$$

$$= \frac{1}{2 X_{eq}} \frac{[S_{\text{max}}^2 X_{eq}^2 + X_{eq}^2]}{S_{\text{max}} X_{eq}}$$

$$\frac{T_{\text{max}}}{T_{\text{st}}} = \frac{\cancel{X_{eq}^2} [1 + S_{\text{max}}^2]}{\cancel{X_{eq}^2} 2 S_{\text{max}}}$$

$$\frac{2.5}{1.75} = \frac{1 + S_{\text{max}}^2}{2 S_{\text{max}}}$$

$$1.75 S_{\text{max}} - 5 S_{\text{max}} + 1.75 = 0$$

Solve for  $S_{\text{max}}$

$$S_{\text{max}} = 2.45 \text{ or } 0.408$$

$$S_{\text{max}} = 0.408$$

$$T_{p.L} = \frac{3 V_{th}^2 R_2' \cdot \cancel{w_s} \left[ \frac{R_2'^2}{S_{p.L}^2} + X_{eq}^2 \right]}{w_s}$$

$$\frac{T_{\text{max}}}{T_{p.L}} = \frac{\cancel{3 V_{th}^2} \cdot \cancel{w_s} X_{eq} \cdot \cancel{w_s} \left[ \frac{R_2'^2}{S_{p.L}^2} + X_{eq}^2 \right] S_{p.L}}{2 \cancel{w_s} X_{eq} \cdot \cancel{3 V_{th}^2} R_2'}$$

$$= \frac{1}{2 X_{eq}} \frac{\left[ \frac{R_2'^2}{S_{p.L}^2} + X_{eq}^2 \right] S_{p.L}}{R_2'}$$

$$= \frac{1}{2 X_{eq}} \frac{\left[ \frac{S_{\text{max}}^2 X_{eq}^2}{S_{p.L}^2} + X_{eq}^2 \right] S_{p.L}}{S_{\text{max}} X_{eq}}$$

$$\frac{T_{\text{max}}}{T_{p.L}} = \frac{S_{\text{max}}^2 + S_{p.L}^2}{2 S_{\text{max}} S_{p.L}}$$

$$2.5 = \frac{(0.408)^2 + S_{p.L}^2}{2 * 0.408 * S_{p.L}}$$

$$S_{FL} = 1.955 \text{ or } 0.085$$

$$S_{FL} = 0.085$$

$$T \propto \frac{I_2^2 R_2'}{S}$$

$$\frac{T_{st}}{T_{FL}} = \frac{I_{2st}^2}{I_{2FL}^2} \cdot \frac{S_{FL}}{S_{st}} = \left( \frac{I_{2st}}{I_{2FL}} \right)^2 \cdot \frac{0.085}{1} = 1.75$$

$$\frac{I_{2st}}{I_{2FL}} = \sqrt{\frac{1.75}{0.085}} = 4.53, \quad I_{2FL} = 1 \text{ p.u.}$$

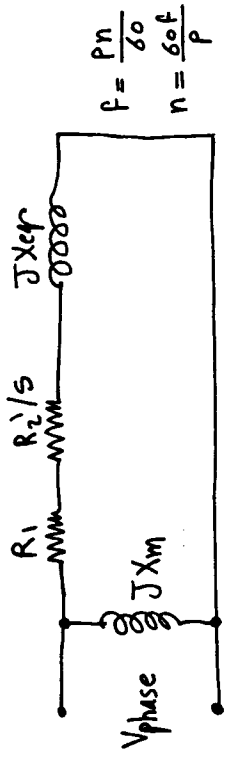
$$I_{2st} = 4.53 \text{ p.u.}$$

$$R_1 = 0.225 \Omega$$

$$X_{eq} = 1.43 \Omega$$

$$R_2' = 0.235 \Omega$$

$$X_M = 31.8 \Omega$$



$$S_{maxT} = \frac{R_2'}{\sqrt{(R_1)^2 + (X_{eq})^2}}$$

$$S_{maxT} = \frac{0.235}{\sqrt{(0.225)^2 + (1.43)^2}} = 0.1623$$

$$T_{max} = \frac{3 V_{ph}^2}{2 \omega_s [R_1 + \sqrt{(R_1)^2 + (X_{eq})^2}]}$$

$$n_s = 600 \text{ r.p.m.}, \quad \omega_s = \frac{2\pi n_s}{60} = 62.8319 \text{ rad/sec}$$

$$T_{max} = \frac{3 * (1270.1706)^2}{125.6637 [0.225 + \sqrt{(0.225)^2 + (1.43)^2}]}$$

$$R_{add} = 1.2126 \text{ } \checkmark 2$$

$$S_{mx1} = 1 = \frac{R_2 + R_{add}}{1.4476}$$

$$T_{mx} = 23027.4203 \text{ N.m.}$$

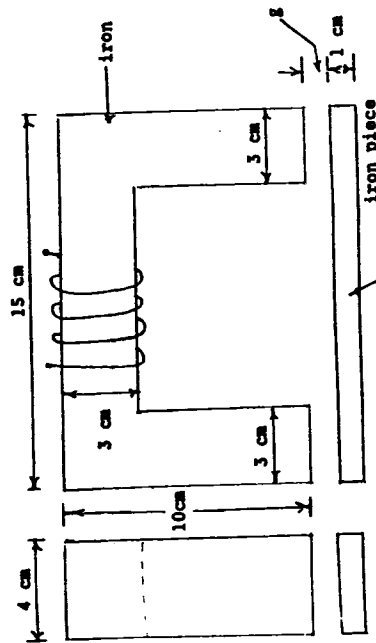
$$T_{mx} = \frac{125.6637 \left[ 0.225 + 1.4476 \right]}{4840000}$$

Instructor: Dr. A. M. El-Serafi  
Note: One sheet of handwritten notes and formulas permitted.

November 1994

Marks

- 25 1. The exciting coil of the shown electromagnet has 1,000 turns and carries a constant current of 5A. Neglecting the leakage, fringing in the air gaps and the reluctance of the magnetic material, calculate:
- The magnetic force acting on the iron piece when the gap length  $g = 1$  cm.
  - The energy supplied by the electrical source if the iron piece is allowed to move from the above position until the air gap length becomes 0.5 cm. Neglect the resistance of the coil.
  - The mechanical work done by the iron pieces for case (b).



- 5 2. How will the magnitude of the magnetic force calculated in (a) of problem (1) be changed:
- If the reluctance of the magnetic material is to be considered?
  - If the fringing flux at the air gaps is not neglected?
  - If the leakage flux is not neglected?

...(2)

- 20 3. A 230-V, 10-hp, 60-Hz, 4-pole, star-connected, 3-phase induction motor has the following per-phase equivalent circuit parameters:

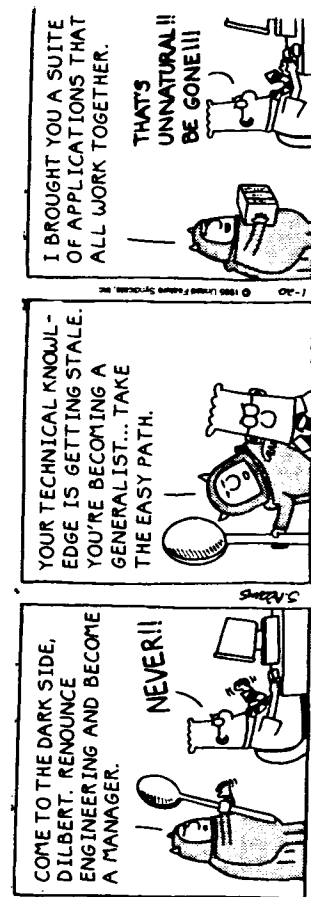
$$r_1 = 0.36\Omega \quad r_2' = 0.19\Omega \quad x_m = 15.5\Omega$$

$$x_1 = 0.47\Omega \quad x_2' = 0.47\Omega$$

Neglecting the core and mechanical losses, calculate:

- The maximum torque of this motor and the speed at which this torque occurs.
- The starting torque of this motor.

\*\*\* The End \*\*\*



- Draw the circle diagram of a 10 hp (7.46 kW), 200 V, 60 Hz, 4-pole, Y-connected, 3-phase slip-ring induction motor with a winding ratio of unity, a stator resistance of  $0.38 \Omega$ /phase and a rotor resistance of  $0.24 \Omega$ /phase. The following are the test readings:  
No-load test: 200 V, 7.7 A,  $\cos \phi_0 = 0.195$   
Locked rotor test: 100 V, 47.6 A,  $\cos \phi_0 = 0.454$

Find:

- The starting torque.
- The maximum torque.
- The slip for maximum torque.
- The maximum power factor.

- A 20 hp, 400 V, 60-Hz, 4-pole, Y-connected, 3-phase squirrel-cage induction motor takes 6 times the full-load current at standstill and rated voltage and develops 1.8 times full-load running torque. The full load current is 30 A.

- What voltage must be applied to produce full-load torque at starting?

- What will be the starting current with this new applied voltage?

- Consider now that this reduced voltage is obtained using an autotransformer. What will be the supply current?

- A 3-phase, 460 V, 1740 r.p.m. 60-Hz, 4-pole wound-rotor induction motor has the following parameters per phase:

$$R_1 = 0.25 \Omega, \quad R_2 = 0.2 \Omega, \quad X_1 = X_2 = 0.5 \Omega, \quad X_m = 30 \Omega$$

The rotational losses are 1700 watts. With the rotor terminals short-circuited, find:

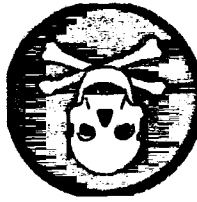
- Starting Torque

- Air-gap power

- Induced torque

- Slip at which maximum torque is developed

- How much external resistance per phase (referred to the stator) should be connected in the rotor circuit so that maximum torque occurs at start?



Instructor: Sherif O. Faried  
Three formula sheets are allowed  
A graph paper is provided

Duration: 3 hours  
December 8, 2001

1. A 200-V, 60-Hz, six-pole, Y-connected, 10-hp (7.46 kW) slip-ring induction motor tested in the laboratory, with the following results:

No load	200 V	7.7 A	520 W
Locked rotor	100	47.6	3743

The effective stator to rotor winding ratio is 1, the stator resistance is 0.38 ohm/phase and the rotor resistance is 0.24 ohm/phase. Draw the motor circle diagram and find:

- Starting torque
- Maximum torque
- Slip for maximum torque
- Maximum power factor
- Maximum output

2. A 10-hp, four-pole, 60-Hz, three-phase induction motor develops its full-load induced torque at 3 per cent slip when operating at 60-Hz and rated voltage. The per-phase circuit model impedances of the motor are:

$R_1 = 0.36 \Omega$	$R_2' = 0.15 \Omega$	$X_m = 15.5 \Omega$
$X_1 = 0.47 \Omega$	$X_2' = 0.47 \Omega$	

Mechanical, core and stray losses may be neglected in this problem. What is the maximum torque of this motor?

3. A 208-V, four-pole, 60-Hz, Y-connected wound rotor induction motor is rated at 15 hp. Its equivalent circuit components referred to the stator winding are:

$R_1 = 0.21 \Omega$	$R_2' = 0.137 \Omega$	$X_m = 13.2 \Omega$
$X_1 = 0.442 \Omega$	$X_2' = 0.442 \Omega$	

$P_{core} = 200 \text{ W}$ ,  $P_{F\&W} = 300 \text{ W}$ . The ratio of stator to rotor turns per phase is 3.5/1.

Due to the requirements of a large starting capability, it is necessary to cause this motor to develop maximum torque at starting. How much external resistance must be added to each rotor phase to meet this requirement?

4. A salient-pole synchronous generator is connected to an infinite bus through an external reactance  $x_e = 0.2 \text{ p.u.}$  (Fig. 1). The synchronous reactances are  $x_d = 1.4 \text{ p.u.}$  and

$x_q = 0.8 \text{ p.u.}$  The generator is supplying the following active and reactive powers to the infinite bus system:  $P_o = 0.9 \text{ p.u.}$ ,  $Q_o = 0$ . The infinite bus voltage is  $V = 1.1 \text{ p.u.}$

Draw the vector diagram and calculate for this operating condition:

- The per-unit terminal and excitation voltages.
- The power angle in degrees.
- The voltage regulation.
- The reluctance power in per-unit.
- The per-unit maximum power the generator can deliver without losing synchronism.



Fig. 1

- A three-phase, Y-connected, round-rotor synchronous motor has a synchronous reactance of 1.0 p.u. and an armature resistance of 0.05 p.u./phase. Do not neglect the armature resistance in your calculations.
  - If the motor takes a line current of 1.0 p.u. at 0.8 p.f. lagging from an infinite bus of 1.0 p.u. voltage, calculate the excitation voltage and the power angle.
  - If the motor is operating on load with a power angle of  $-21.1233$  degrees and the excitation is so adjusted that the excitation voltage is equal to 1.6481 p.u., determine the armature current and the power factor of the motor.
- A 13.8 kV, 10 MVA, 60-Hz, 2-pole, Y-connected turbine-generator has a synchronous reactance of 22.8528 ohm/phase and a negligible armature resistance. This generator is operating in parallel with a very large power system with a voltage magnitude of 13.8 kV.
  - What is the magnitude of the excitation voltage (in p.u.) at rated current and 0.8 p.f. lagging.
  - What is the power angle of the generator under the conditions of (a)
  - If the field current is constant, what is the maximum power (in p.u.) possible out of this generator?
  - At the absolute maximum power possible, how much reactive power (in p.u.) will this generator be supplying or consuming? Sketch the corresponding phasor diagram.
- A three-phase synchronous generator is operating at a lagging power factor condition on an infinite bus. Treat the machine as lossless. If the prime mover power supplied to the generator is increased, but the field current is adjusted so that the output reactive power is unchanged, draw the vector diagram and qualitatively describe the changes in  $I_a$ ,  $E_f$ ,  $\phi$  and  $\delta$ .



Duration: 3 hours  
December 2000

Instructor: Sherif O. Faried  
A one formula sheet is allowed  
A graph paper is provided

1. Prove that if the stator resistance of a three-phase induction motor is neglected ( $R_1 = 0$ ), the torque/slip curve of such a motor can be expressed by the relation:

$$\frac{T}{T_{\max}} = \frac{\frac{s}{s_{\max T}} + \frac{s_{\max T}}{s}}{2}$$

where  $s$  and  $s_{\max T}$  are the slips corresponding to  $T$  and  $T_{\max}$  respectively.

2. The approximate per-phase equivalent circuit for a 3-phase, 4-pole, 60-Hz, 1710 rpm double-cage rotor induction machine is shown in Fig. 1. The standstill rotor impedances referred to the stator are as follows:

Outer cage:  $4.0 + j1.5 \Omega$

Inner cage:  $0.5 + j4.5 \Omega$

If the stator impedance is neglected,

- (a) Determine the ratio of currents in the outer and inner cages for standstill and full-load conditions.

- (b) Determine the starting torque of the motor as percent of full-load torque.

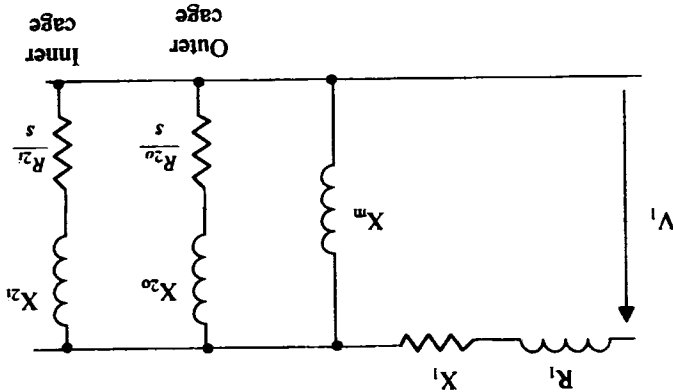


Fig. 1

3. A salient-pole synchronous generator supplies a load at a unity power factor to an infinite bus whose voltage is 1.05 p.u. The generator e.m.f ( $E_f$ ) under this condition is 1.4 p.u. If  $X_d = 0.95$  p.u. and  $X_q = 0.65$  p.u.
  - (a) Draw the vector diagram under this operating condition.
  - (b) Calculate the power delivered to the infinite bus and the load angle.

4. The following data are obtained from the open-circuit and short-circuit characteristics of a three-phase, wye-connected, four-pole, 150-MW, 0.9-p.f., 12.6-kV, 60-Hz, hydrogen-cooled turbine-generator with negligible armature resistance:

Open-circuit characteristic									
Field current, A	200	300	400	500	600	700	800	900	1150
Line-to-line voltage, kV	3.8	5.8	7.8	9.8	11.3	12.6	13.5	14.2	15.58
Short-circuit characteristic									
Field current, A	350								
Armature current, A	4043								
	700								
	8086								

Determine:

- The unsaturated synchronous reactance in p.u.
- The saturated synchronous reactance in p.u. and the short-circuit ratio.
- The estimated field current and voltage regulation for rated voltage, rated current and a unity p.f. operation.
- The power angle under this condition.

5. A salient-pole synchronous generator is connected to an infinite bus through an external reactance  $x_e = 0.2 \text{ p.u.}$  (Fig. 2). The synchronous reactances are  $x_d = 1.4 \text{ p.u.}$  and  $x_q = 0.8 \text{ p.u.}$  The generator is supplying the following active and reactive powers to the infinite bus system:  $P_o = 0.9 \text{ p.u.}$ ,  $Q_o = 0.3 \text{ p.u.}$  The infinite bus voltage is  $V = 1 \text{ p.u.}$

Calculate for this operating condition:

- The per-unit terminal and excitation voltages.
- The power angle in degrees.
- The voltage regulation.
- The reluctance power in per-unit.
- The per-unit maximum power the generator can deliver without losing synchronism.
- $P_G$  and  $Q_G$  in per-unit.

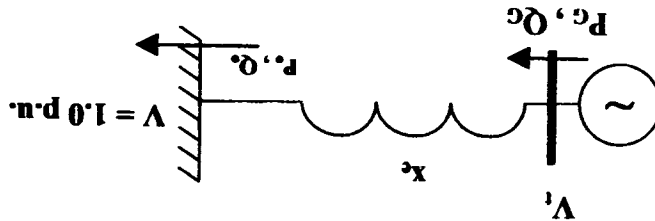
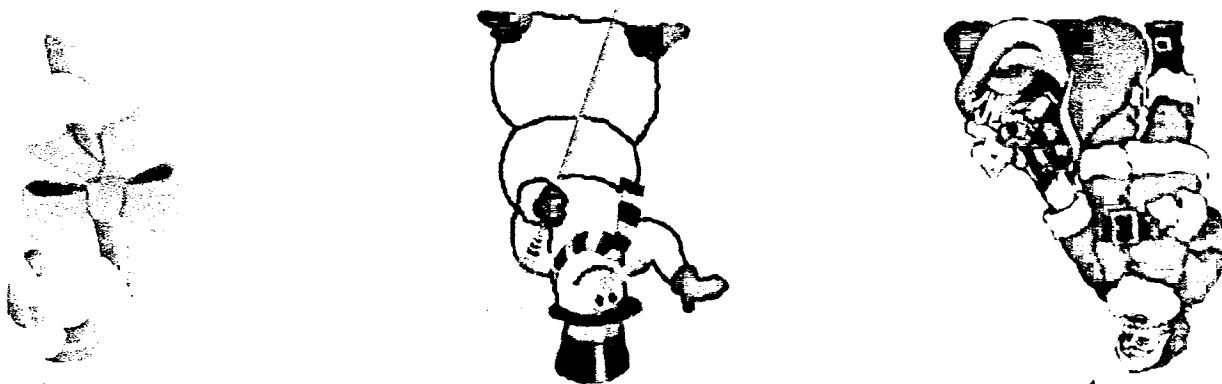


Fig. 2

6. A three-phase, Y-connected, round rotor synchronous motor has a synchronous reactance of 1.2 p.u. and negligible armature resistance.
- If the motor takes a line current of 0.9 p.u. at 0.85 p.f. leading from an infinite bus of 1.0 p.u. voltage, draw the vector diagram and calculate the excitation voltage and the power angle.

(b) If the motor is operating on load with a power angle of  $-30^\circ$ , and the excitation is adjusted that the excitation voltage is equal in magnitude to the terminal voltage, determine the active and reactive power delivered to the motor.

# MERRY CHRISTMAS



Instructor: S.O. Faried  
Duration: 3 Hours

December 16, 1997

1. (a) The torque expression of a three-phase induction motor can be given by:

$$T = \frac{3V_n^2 R_2' / s}{\omega_s [(R_n + R_2' / s)^2 + (X_n + X_2')^2]}$$

Show that in the limit of negligible armature resistance  $R_1$ , this expression can be written as

$$T = \frac{2 T_{max}}{\frac{s_{max}}{s} + \frac{s}{s_{max}}}$$

where  $T_{max}$  is the maximum torque and  $s_{max}$  is the slip at maximum torque.

- (b) A 230-V, 4-pole, 10-hp, 60-Hz, three-phase induction motor has the following per-phase equivalent circuit parameters:

$$\begin{aligned} R_1 &= 0.0 \Omega & R_2' &= 0.332 \Omega \\ X_1 &= 1.1 \Omega & X_2' &= 0.47 \Omega \\ X_m &= 26 \Omega \end{aligned}$$

- i) What is the maximum torque of this motor? At what speed and slip does it occur?  
ii) What is the starting torque of this motor?

2. A 100-MVA, 11.8 kV, 60-Hz, 2-pole, Y-Connected, synchronous generator has a per-unit synchronous reactance of 0.8 and a negligible armature resistance. The generator is connected to an infinite bus system of 1.0 p.u. voltage through a tie-line of 0.2 p.u. reactance.

- (a) If the generator is delivering its full-load current at 0.8 P.F. lagging to the infinite bus, find:

- i) the terminal voltage  $V_t$   
ii) the excitation voltage  $E_f$   
iii) the generator power angle  $\delta$   
iv) the voltage regulation.

- (b) If the generator excitation is adjusted such that the magnitude of the terminal voltage  $V_t$  is equal to the infinite bus voltage while the generator is still delivering its full-load current, draw the system vector diagram and find:

- i) the power factor at the infinite bus  
ii) the excitation voltage  $E_f$   
iii) the generator power angle  $\delta$   
iv) the maximum power that can be delivered without losing synchronism.

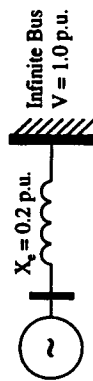


Figure 1

3. (a) Starting from the steady-state power-angle equation of a salient-pole synchronous machine with negligible armature resistance and fixed field excitation, show that the condition for maximum power is given by:

$$\cos \delta = -K + \sqrt{K^2 + 0.5}$$

where

$$K = \frac{E_f X_q}{4(X_d - X_q)W}$$

- (b) The direct- and quadrature-axis synchronous reactances of a salient-pole synchronous generator are  $X_d = 1.0$  p.u. and  $X_q = 0.8$  p.u. The generator is connected to an infinite bus of 1.0 p.u. voltage.

- i) If the machine loses synchronism when the power angle is  $81.414473^\circ$ , what is the p.u. excitation voltage at pullout?  
ii) For the case described in (i), what are the corresponding active and reactive powers?

4. In the two-machine system shown in Figure 2, the excitations of the two machines are so controlled that the terminal voltages of the two machines remain constant and equal to 1.0 p.u.

- (a) Derive an expression for the power fed from the synchronous generator to the synchronous motor as a function of their terminal voltages  $V_g$  and  $V_m$  and the angle between the quadrature axes of the two machines, ( $\delta$ ).

- (b) What will be the maximum power which can be fed without losing synchronism?

- (c) What is the value of  $\delta$  in this case?

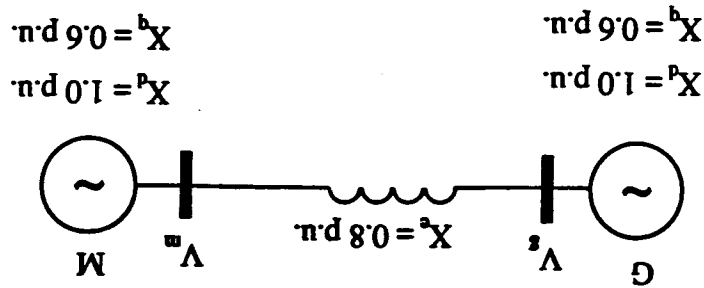


Figure 2

5. In the system shown in Fig. 3, one circuit of the double-circuit transmission line is opened suddenly. The system parameters and operating conditions before the disturbance are indicated in the same figure. Using the equal-area criterion, check the transient stability of the system after this disturbance. If it is stable, find the maximum angle of swing.

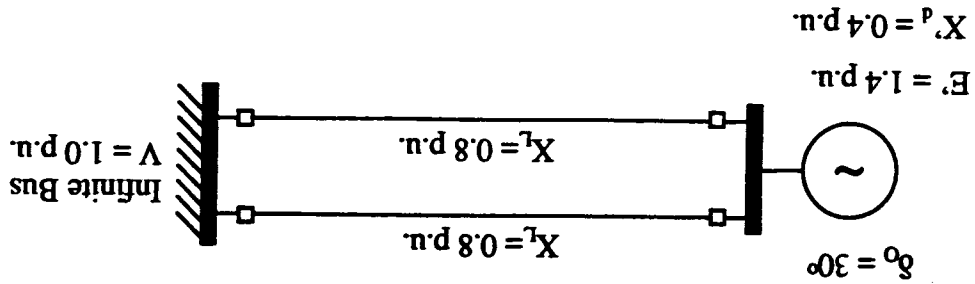


Figure 3

... The End ...

11/5  
EE461 Midterm

2. Find the difference equation for a system that has output

$$y(n) = 0.25^n(u(n) + 0.75u(n-1)) + 0.75^n u(n)$$

when the input is

$$x(n) = 0.25^n u(n) \quad \text{[m assuming this is } u(n) \text{ so that I can complete the question (even though it will be wrong)]}$$

$$y[n] = 0.25^n u[n] + 0.75 \cdot 0.25^n u[n-1] + 0.75^n u[n]$$

$$Y(z) = \frac{1}{1-0.25z^{-1}} + \frac{0.75X}{1-0.25z^{-1}} + \frac{1}{1-0.75z^{-1}}$$

$$= \frac{1.75(1-0.75z^{-1}) + 1-0.25z^{-1}}{(1-0.25z^{-1})(1-0.75z^{-1})} = \frac{1.75 + 1 + 1.3z^{-1} - 0.75z^{-1}}{(1-0.25z^{-1})(1-0.75z^{-1})}$$

$$= \frac{2.75 + 1.06z^{-1}}{(1-0.25z^{-1})(1-0.75z^{-1})}$$

$$x[n] = 0.25^n u[n]$$

$$X(z) = \frac{1}{1-0.25z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2.75 + 1.06z^{-1}}{(1-0.25z^{-1})(1-0.75z^{-1})} = \frac{2.75 + 1.06z^{-1}}{1-0.75z^{-1}}$$

$$Y(z)(1-0.25z^{-1}) = X(z)(2.75 + 1.06z^{-1})$$

$$y[n] - 0.25y[n-1] = 2.75x[n] + 1.06x[n-1]$$

Date: Wednesday, October 9, 2002

Time = 1 hour 30 minutes

Text Books and Notes Only - no worked examples or solved problems

1. An engineer is to design a NCO that has a frequency resolution of less than  $10^{-6}$  radians/sample (i.e. the frequency can be incremented in steps of  $\Delta\omega$ , where  $\Delta\omega < 10^{-6}$  radians/sample) and an SNR of greater than 50 dB on the output sinusoid.

(a) What is the minimum size that can be used for the phase accumulator?

(b) What is the minimum size ROM (LUT) that can be used? Specify the size in number of bits.

d)  $\Delta\omega < 10^{-6}$  rad/sample, SNR > 50dB

$\Delta F < 1.6 \times 10^{-6}$  cycles/sample

The number of bits in the P.A., N, should obey:

$$\frac{1}{2^N} < 1.6 \times 10^{-6} \text{ cycles/sample}$$

for  $N = 20, \frac{1}{2^{20}} = 9.54 \times 10^{-7}$ , so select  $N = 20$

b) Find  $N_R$  and  $N_A$  such that SNR > 50dB

$N_D$	$N_A$	SNR
11	11	60.25 dB
10	11	58.48 dB
10	10	54.23 dB
9	10	52.46 dB
9	9	48.21 dB
10	9	48.80 dB
9	10	48.76 dB

\* 9 10 52.46dB ← Best Combination corresponds to optimal of  $N_A = N_D + 1$

total # of bits in Rom = # address > # bits/address

$$= 2^{10} \cdot 9$$

$$= 9261 \text{ bits}$$

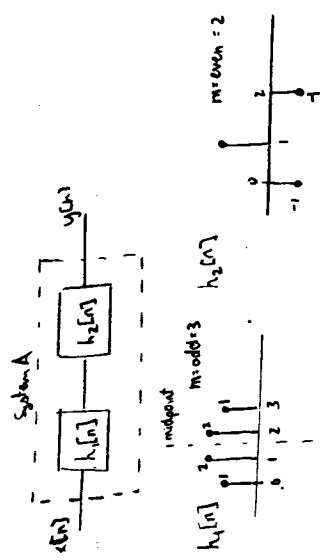
4. A system, say system A, is composed of two systems in tandem (cascade). The two systems in tandem (cascade) have impulse responses

$$h_1(n) = \delta(n) + 2\delta(n-1) + 2\delta(n-2) + \delta(n-3)$$

and

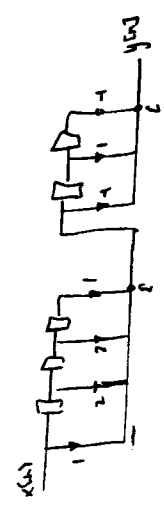
$$h_2(n) = -\delta(n) + \delta(n-1) - \delta(n-2)$$

Find an expression for the phase response of system A. (i.e. find  $\angle H_A(e^{j\omega})$ )



- both responses are symmetric.  
- both subplots are FIR

- System A will have a Symmetric Response



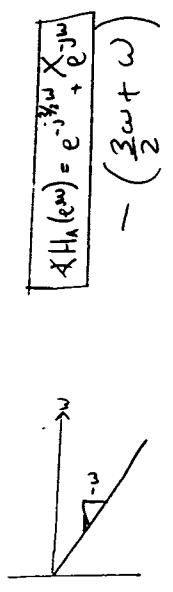
$h_1(n)$  is a linear phase system:  $H_1(e^{j\omega}) = H_1(e^{j\omega})$

$$H_1(e^{j\omega}) = e^{-j\frac{3}{2}\omega} A(\omega) \times H_2(e^{j\omega})$$

$$\angle H_1(e^{j\omega}) = \angle e^{-j\frac{3}{2}\omega} = -\frac{3}{2}\omega$$

$h_2(n)$  is a linear phase system:  $H_2(e^{j\omega}) = H_2(e^{j\omega})$

$$\angle H_2(e^{j\omega}) = \angle e^{-j\omega} = -\omega$$



3. Find the impulse response if the system function is

$$H(z) = \frac{1+j1}{1-j0.5z^{-1}} + \frac{1-j1}{1+j0.5z^{-1}}$$

- need to find an acceptable form to take inverse z transform

$$H(z) = \frac{(1+j)(1+j0.5z^{-1}) + (1-j)(1-j0.5z^{-1})}{(1-j0.5z^{-1})(1+j0.5z^{-1})}$$

$$= \frac{1+j0.5z^{-1} - 0.5z^{-1} + 1 - j0.5z^{-1} - 0.5z^{-1} + 0.25z^{-1}}{1 - 0.25z^{-1}}$$

$$= \frac{2 - j2z^{-1}}{1 + 0.25z^{-1}}$$

$$H(z) = \frac{2}{1+0.25z^{-1}} - \frac{2z^{-1}}{1+0.25z^{-1}}$$

delay of 1 sample

$$h[n] = 2(0.25)^n u[n] - 2(0.25)^{n-1} u[n-1]$$

$$h[n] = 2(0.25)^n u[n] - 2(0.25)^{n-1} u[n-1]$$

# EE461 Midterm

NAME: [REDACTED]

STUDENT NO.: [REDACTED]

Date: Wednesday, November 20, 2002

Time = 1 hour 30 minutes

Text Books and Notes Only

Absolutely no worked examples or solved problems

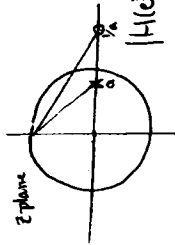
1.	4
2.	5
3.	3 1/2
4.	5
5.	5
6.	1
TOTAL	23 1/2

5. Consider a causal linear time-invariant system with system function

$$H(z) = \frac{1 - a^{-1}z^{-1}}{1 - az^{-1}}$$

- (1) (a) What is  $|H(e^{j\omega})|$  at frequencies  $\omega = 0$ ,  $\omega = \pi/2$ , and  $\omega = 7\pi/8$  radians per sample?
- (2) (b) Write the difference equation that relates the input and the output of the system.
- (1) (c) for what range of  $a$  is the system stable?

For a real system, pole must be inside unit circle implying that the zero is closer to origin



$$\begin{aligned} |H(e^{j0})| &= \frac{1-a^{-1}}{1-a} \\ |H(e^{j\pi/2})| &= \frac{\sqrt{\frac{1}{a^2} + 1}}{\sqrt{a^2 + 1}} \quad \text{graphical method} \\ |H(e^{j7\pi/8})| &= \frac{1 - \frac{1}{a} e^{-j7\pi/8}}{1 - a e^{j7\pi/8}} \end{aligned}$$

$$\begin{aligned} &= \frac{1 - \frac{1}{a} (\cos 7\pi/8 - j \sin 7\pi/8)}{1 - a (\cos 7\pi/8 + j \sin 7\pi/8)} \\ &= \frac{1 - \frac{1}{a} \cos 7\pi/8 + \frac{1}{a} j \sin 7\pi/8}{1 - a \cos 7\pi/8 - a j \sin 7\pi/8} \end{aligned}$$

$$b) H(z) = \frac{1 - a^{-1}z^{-1}}{1 - az^{-1}} = \frac{X(z)}{Y(z)} \quad X(z) = Y(z)$$

$$Y(z) - Y(z)a^{-1}z^{-1} = X(z) - X(z)a^{-1}z^{-1}$$

$$y[n] - a^{-1}y[n-1] = x[n] - a^{-1}x[n-1]$$

c) the pole must be inside the Unit Circle

$$0 < |a| < 1$$

this represents an all pass system



(5)

2. A system has a finite impulse response of length 5 (i.e.  $M=4$ ). When an input of  $\sqrt{2} \cos(\frac{\pi}{4}n)$  is applied, the output for  $n = 0, 1, \dots, 5$ , is the real sequence  $\{1.4, 3.8, -12.1, -6.8, -42.84, -23.828\}$ . When an input of  $\sqrt{2} \sin(\frac{\pi}{4}n)$  is applied, the output for  $n = 0, 1, \dots, 5$ , is the real sequence  $\{0, 1, 3.4, -6.2, -9.142, -36.757\}$ . What is the frequency response of the system at  $\omega = \pi/4$  radians per sample?

$$H(e^{j\pi/4}) = y[n]e^{-j\pi/4} \quad \text{for } x[n] = e^{j\pi/4}$$

$$x[n] = \sqrt{2} \cos(\frac{\pi}{4}n)$$

$$x[n] = \sqrt{2} \sin(\frac{\pi}{4}n)$$

$$\text{for } x[n] = e^{j\pi/4}$$

$$\text{then } x[n] = (x[n] + jx[n]) \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \{1, 2.7 + j0.707, -9.56 + j2.404, -4.808 - j4.26, -30.3 - j6.46, -16.85 - j26\}$$

$$H(e^{j\pi/4}) = (-16.85 - j26) e^{-j\pi/4}$$

$$|H(e^{j\pi/4})| = 31 \angle 10.567^\circ$$

$$H(e^{j\pi/4}) = y[n]e^{-j\pi/4} = y[4]$$

$$H(e^{j\pi/4}) = \frac{-42.84 + j23.828}{\sqrt{2}} + \frac{j0.707 - 9.142}{\sqrt{2}}$$

$$= 31 e^{-j2.93}$$

$$\therefore 31 \angle -168^\circ$$

(7)

1. Please circle the correct answer for the questions that follow. Note that wrong answers will be subtracted from the right answers. All parts are worth the same.

The questions are based on three discrete time systems, each with system functions containing only zeros. System 1 has 6 zeros located at  $z = 0.7e^{j\pi/4}, 0.7e^{-j\pi/4}, 1, -1, .5$  and 2. System 2 has 22 zeros at  $z = c_k$ , where  $c_k = e^{j\frac{2\pi k}{23}}$ ,  $k = 2, 3, \dots, 23$ . System 3 has 17 zeros, with 4 at  $z = 1, 3$  at  $z = -1, 5$  at  $z = e^{j\pi/4}$  and 5 at  $z = e^{-j\pi/4}$ .

(a) The impulse response of system 1 is

a) symmetric, b) antisymmetric, c) neither symmetric nor antisymmetric about its midpoint.

(b) The impulse response of system 2 is

a) symmetric, b) antisymmetric, c) neither symmetric nor antisymmetric about its midpoint.

(c) The impulse response of system 3 is

a) symmetric, b) antisymmetric, c) neither symmetric nor antisymmetric about its midpoint.

(d) The magnitude of the frequency response of system 3 is greater at  $\omega = \pi/4$  radians/sample of b)  $\omega = 3\pi/4$  radians/sample.

(e) The magnitude of the frequency response of system 2 is

a) zero, b) not zero at  $\omega = \pi$  radians/sample.

(f) The magnitude of the frequency response of system 1 is

a) zero, b) not zero at  $\omega = 0.5$  radians/sample.

(g) The phase of the frequency response of system 2 at  $\omega = \pi/10$  radians per sample (i.e. angle of  $H_2(e^{j\pi/10})$  is

a)  $-17\pi/20$  radians b)  $-27\pi/20$  radians c) neither a) nor b)

- (5) 4. A digital filter is constructed by sampling the impulse response of an analog filter with a sampling rate of 1000 samples/second. Find an expression for the frequency response of the digital filter if the analog filter has system function

$$H_a(s) = \frac{s+7}{(s+3)(s+2)}$$

$$F = 1000 \text{ samples/sec} \quad T_d = \frac{1}{1000} \text{ sec/sample}$$

→ Sampling impulse response is impulse invariant

$$H_d(s) = \frac{A}{s+3} + \frac{B}{s+2}$$

$$A = \frac{s+7}{s+2} \Big|_{s=-3} = \frac{4}{-1} = -4$$

$$B = \frac{s+7}{s+3} \Big|_{s=-2} = \frac{5}{1} = 5$$

$$H_d(s) = \frac{5}{s+2} - \frac{4}{s+3}$$

→ assuming  $h[n] = T_d h_a(nT_d)$

$$\text{then: } H(z) = T_d \frac{5}{1 - e^{-2T_d} z^{-1}} - T_d \frac{4}{1 - e^{-3T_d} z^{-1}}$$

$$H(z) = T_d \left( \frac{5(1 - e^{-3T_d} z^{-1}) - 4(1 - e^{-2T_d} z^{-1})}{1 - (e^{-2T_d} + e^{-3T_d})z^{-1} + e^{-(2T_d+3T_d)}z^{-2}} \right)$$

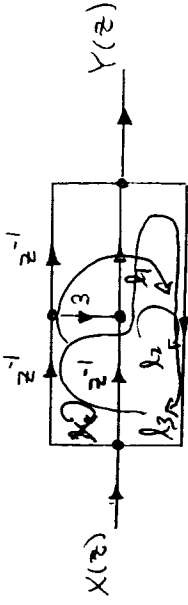
$$H(z) = T_d \left( \frac{1 - 5e^{-3T_d} z^{-1} + 4e^{-2T_d} z^{-1}}{1 - e^{-2T_d} + e^{-3T_d} z^{-1} + e^{-(2T_d+3T_d)}z^{-2}} \right)$$

$$H(z) = 0.001 \left( \frac{1 + (4e^{-0.002} - 5e^{-0.003})z^{-1}}{1 - (e^{-0.002} + e^{-0.003})z^{-1} + e^{-0.005}z^{-2}} \right)$$

$$H(e^{j\omega}) = 0.001 \left( \frac{1 + (4e^{-0.002} - 5e^{-0.003})e^{-j\omega}}{1 - (e^{-0.002} + e^{-0.003})e^{-j\omega} + e^{-0.005}e^{-j2\omega}} \right)$$

$$H(e^{j\omega}) \approx 0.001 \left( \frac{1 - 1e^{-j\omega}}{1 - 2e^{-j\omega} + e^{-j2\omega}} \right)$$

- (5) 3. Redraw the graph below in direct form 2 structure. Show all the coefficients on the direct form 2 graph.



Simplify:

$$P_1 = z^{-1} \quad l_1 = z^{-2}$$

$$P_2 = z^{-2} \quad l_2 = z^{-1}$$

$$P_3 = 3z^{-1} \quad l_3 = 3z^{-1}$$

$$\Delta = 1 - (3z^{-1} + z^{-1})z^{-2}$$

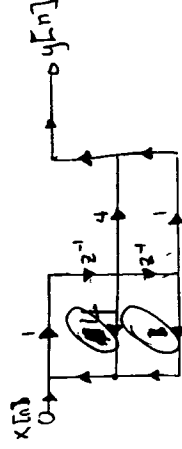
$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$\Delta_3 = 1$$

$$H(z) = \frac{\sum P_i \Delta_i}{\Delta} = \frac{z^{-1} + z^{-2} + 3z^{-1}}{1 - z^{-2} - 4z^{-3}}$$

$$H(z) = \frac{4z^{-1} + z^{-2}}{1 - z^{-2} - 4z^{-3}}$$



6. Find an expression for the coefficients,  $b_k$ ,  $k = 0, 1, \dots, M$ , for a symmetric linear phase filter of length  $M + 1$ , where  $M$  is even, that best approximates an ideal bandpass magnitude response, with the pass band between  $\omega_L$  and  $\omega_H$ .



$$|H(e^{j\omega})| = \begin{cases} 1, & \omega_L < \omega < \omega_H \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \frac{1}{2\pi} \int_{\omega_L}^{\omega_H} \sin(\omega(n - M/2)) d\omega$$

$$h[n] = \begin{cases} 0 & n = M/2 \\ \frac{1}{2\pi} \cdot \frac{1}{(n - M/2)} [\cos(\omega_L(n - M/2)) - \cos(\omega_H(n - M/2))] & n \neq M/2 \end{cases}$$

5. A digital filter is constructed by a bilinear transformation on an analog filter with a sampling rate of 1000 samples/second. Find an expression for the frequency response of the digital filter if the analog filter has system function

$$H_a(s) = \frac{s + 7}{(s + 3)(s + 2)}$$

$$H(z) = H_a(s) \Big|_{s = \frac{2}{T} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right)}$$

$T = \frac{1}{1000} \text{ samples}$

$$H(z) = \frac{\frac{2}{T} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right) + 7}{\left[ \frac{\frac{2}{T} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right) \right]^2 + 5 \left[ \frac{\frac{2}{T} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right) \right] + 6} \cdot \frac{[T(1 + z^{-1})]^2}{[T(1 + z^{-1})]^2}}$$

$$= \frac{2(1 - z^{-1})[T(1 + z^{-1})] + 7[T(1 + z^{-1})]^2}{[2(1 - z^{-1})]^2 + 5[2(1 - z^{-1})][T(1 + z^{-1})] + 6[T(1 + z^{-1})]^2}$$

$$= \frac{2T(1 - z^{-2}) + 7T^2(1 + 2z^{-1} + z^{-2})}{4(1 - 2z^{-1} + z^{-2}) + 10T(1 - z^{-2}) + 6T^2(1 + 2z^{-1} + z^{-2})}$$

$$= \frac{(2T + 7T^2) + 14Tz^{-1} + (7T^2 - 2T)z^{-2}}{(4 + 10T + 6T^2) + (12T^2 - 8)z^{-1} + (6T^2 + 10T + 4)z^{-2}}$$

$$H(e^{j\omega}) = \frac{(2T + 7T^2) + (14T)e^{j\omega} + (7T^2 - 2T)e^{j2\omega}}{(4 + 10T + 6T^2) + (12T^2 - 8)e^{j\omega} + (6T^2 + 10T + 4)e^{j2\omega}}$$

with  $T = 0.001$

$$H(e^{j\omega}) = \frac{0.002007 + 0.000014e^{j\omega} - 0.001993e^{j2\omega}}{4.000006 - 7.999999e^{j\omega} + 3.999996e^{j2\omega}}$$

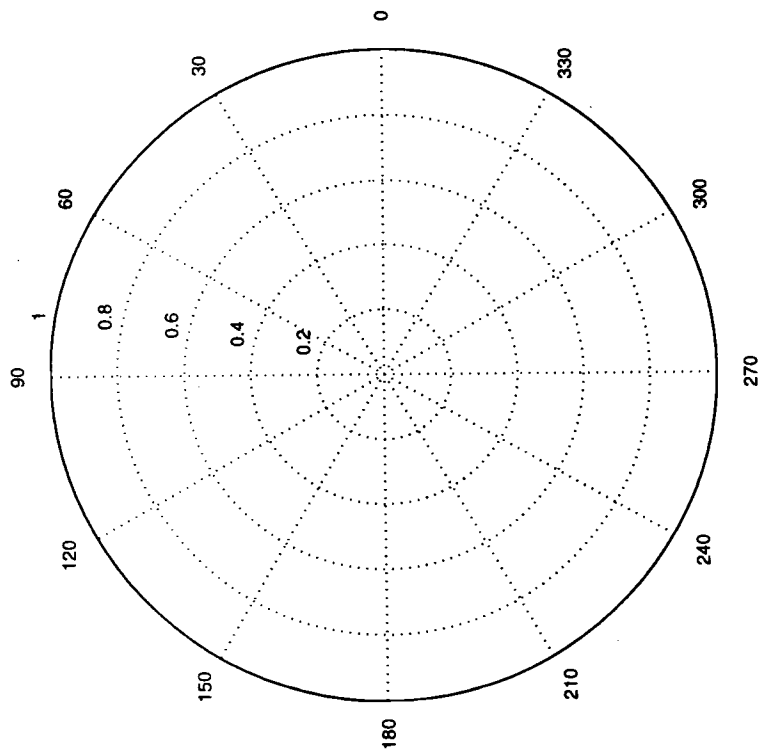
# EE484 MIDTERM 2

Thursday, March 22, 2001

Time - 1 hour.

Only two formula sheets allowed.

All Questions worth 5



1. A bilinear transformation is used to transform continuous-time system function

$$H_c(s) = \frac{0.02}{s^2 + 0.2s + 0.02}$$

to discrete-time system function  $H(z)$ .

- (a) Find the poles and zeros of  $H(z)$ . (NOTE: Be careful as the answers to parts b) and c) depend on this answer being correct.)
- (b) Is this a low-pass, band-pass or high-pass filter? (To obtain credit you must justify your answer.)
- (c) Is there ripple in the stopband? (To obtain credit you must justify your answer.)

2. An junior engineer is asked to design a digital band-pass filter by applying a bilinear transformation to an analog band-pass filter. The digital filter is specified as follows:

$$1 - 0.1 < |H(e^{j\omega})| < 1 + 0.1; \quad 0 \leq \omega < \frac{\pi}{4}$$

$$|H(e^{j\omega})| < 0.01; \quad \frac{\pi}{2} \leq \omega \leq \pi$$

Specify the analog filter that has to be designed.

3. Find the order and parameter  $\Omega_c$  for a low pass Butterworth filter that satisfies:

$$0.9 \leq H_c(j\Omega) \leq 1; \quad 0 \leq \Omega \leq \frac{\pi}{4}$$

$$H_c(j\Omega) \leq 0.01; \quad \frac{\pi}{2} \leq \Omega \leq \pi$$

From  
mid #1  
2001

The following code is for a TMS320C31 DSP chip, mounted on a board similar, though not identical to the board used in the class project. Assume the initial *asm* include file does all the required initialisation for the board, including setting the sampling rate and configuring the D/A and A/D chip. Valid memory addresses from 0403000h to 0403007h.

```

=====
10 University of Southern California
10 M800 Digital Signal Processing Course
10 TMS320C31 Program
=====
1 .sect ".text", 0x000070
2 .start "start", 0x000010
3 .start "start", 0x000010
4 .start "start", 0x000010
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6 .start "start", 0x000010
7 .sect ".text", 0x000010
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81 .sect ".text", 0x000010
82 .sect ".text", 0x000010
83 .sect ".text", 0x000010
84 .sect ".text", 0x000010
85 .sect ".text", 0x000010
86 .sect ".text", 0x000010
87 .sect ".text", 0x000010
88 .sect ".text", 0x000010
89 .sect ".text", 0x000010
90 .sect ".text", 0x000010
91 .sect ".text", 0x000010
92 .sect ".text", 0x000010
93 .sect ".text", 0x000010
94 .sect ".text", 0x000010
95 .sect ".text", 0x000010
96 .sect ".text", 0x000010
97 .sect ".text", 0x000010
98 .sect ".text", 0x000010
99 .sect ".text", 0x000010
100 .sect ".text", 0x000010
=====

```

**For the filter implemented by the above program:**

- Determine the order of the filter.
- Determine the impulse response, the transfer function and the difference equation for the filter.
- Is this an IIR or FIR filter? Is this a lowpass/highpass/bandpass or bandstop filter?
- What are the addresses at which the D/A and A/D converter are found? Determine the number of bits of resolution for A/D and D/A converter and their position in the D/A and A/D register.
- Lines 10 and 11 are replaced as follows

```

10 LDI CHEN3,AR1
11 LDI CHEN4,ARO ;

```

**Determine appropriate values for lines 4,5,29,32,33. If an input sequence given below is applied**

$$x(n) = \{1.5, 2, 3, 4, 0, 0, 0, 0\} \quad (1)$$

**Determine the output of the filter to this input sequence. What type of filter is implemented by this code?**

- f) What is the gain of the filter? Show how to change the gain of the filter to 2.0.

## 2 Question #2

**The following output was generated by Matlab**

```
f = [-0.1 -2.8 -6.5 -6.7 -9.1]
Colonne 1 through 7
           0.1000    0.2000    0.3000    0.4000    0.5000
Colonne 8 through 12
          -0.0000    0.7000    0.9000    0.9000    1.0000
m=[0 0 0 1 1 1 0 0 0 ]
n=[0 0 0 0 0 0 1 1 1 1 0 0 0 0 ]
(f,a) = polyval(m,f,a)
           -0.1104   -0.0046   -0.1818    0.0009    0.1822    0.0041   -0.1101
           1.0000   -0.2374
           0.0000   -0.1866    0.0819   -0.0458   -0.0242    0.0299
           -0.0037 + 0.0099i
           -0.0037 - 0.0099i
           0.0036 + 0.0094i
           0.0036 - 0.0094i
           -0.0043
           0.0043
           -0.0037 + 0.0099i
           -0.0037 - 0.0099i
           -0.0036 + 0.0100i
           -0.0036 - 0.0100i
           0.0018 + 0.0060i
           0.0018 - 0.0060i
           0.1104
```

- Is this an IIR or FIR filter? Is this a lowpass/highpass/bandpass or bandstop filter?
- Determine the order, the transfer function and the difference equation for this filter.

c) Plot the poles and zeros of the filter.

**The following is another set of output from Matlab**

```

x=[10.1.2.3.4.06.8.6.7.0.0.1]
x
Columns 1 through 7
    10.0000    0.1000    0.2000    0.3000    0.4000    0.4000    0.6000
    8.0000    0.7000    0.8000    0.9000    1.0000
m=[1 1 1 1 1.707 0 0 0 0]
m
Columns 1 through 7
    1.0000    1.0000    1.0000    1.0000    1.0000    1.7070    0
Columns 8 through 12
    0
b = roots(x,m)
b
    =
    -0.0849    0.4002    0.4002    0.4002
DL_m = diff(b,1)
DL_m
    =
    -0.0849    0.4851    0.0000    0.0000

```

```

E_u =
    0.9004
    0.9012 - 0.90121
    0.0000 - 1.02861
    -0.8021 - 0.90111
    -0.8004
    -0.0088
    -0.0000 - 0.00881
    -0.0000 - 0.11721
    -0.2280 - 0.22801

v =
    0
    0.2677
    0.7764
    1.1781
    1.8708
    2.1828
    2.7408
    3.1408

E_u = abs(E_u)
    1.9004
    1.7748
    1.0286
    0.9011
    0.7101
    0.8004
    0.1172
    0.2280

E_u = max(abs(E_u)) / (p_u(100))
    0
    -46.0000
    -90.0000
    -180.0000
    -180.0000
    -226.0000
    -90.0000
    -126.0000

```

- Is this an IIR or FIR filter? Is this a lowpass/highpass/bandpass or bandstop filter? What is the order of this filter?
- If the sampling frequency is  $4\text{ kHz}$ , determine the cutoff frequency.
- Plot the magnitude and phase response of the filter.
- Determine the transfer function and difference equation of this filter.

### 3 Question #3

Discovered in the basement archives of a Nashville recording studio is an unreleased original, early recording by Elvis. It seems as if the recording was discarded due to significant corruption. The recording is corrupted by harmonic distortion that is given by

$$D(\omega) = 0.5^k \cos(2\pi f_0 k)$$

for  $f_0 = 1\text{KHz}$  and  $k = 1, 2, 3, 4, 5, 6$ .

- Design a comb filter that will remove this distortion. Specify the transfer function, difference equation and sampling rate.
- After digitally processing the recording, it was played for a studio executive, who was not satisfied with the results. Further analysis indicates that a cascade of three notch filters, to remove the first three harmonics, will provide better results. The sampling rate is specified as 16KHz. Each of the notch filters is to have a 3dB bandwidth of 50Hz. Determine the transfer function and difference equation for the notch filter that will remove the 1KHz distortion. Assume each notch filter can be designed independently.

6

DO ANY TWO OF THE FOLLOWING FOUR QUESTIONS  
IE Answer any two questions out of questions 4,5,6 and 7.

### 4 Question #4

Design a Lowpass filter using the Frequency Sampling Method.

- Determine the coefficients of a linear-phase FIR filter of length  $M = 15$  which has a symmetric unit sample response and a frequency response that satisfies:

$$H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1 & k = 0, 1, 2, 3, 4 \\ 0.3927 & k = 5 \\ 0 & k = 6, 7 \end{cases}$$

- Plot the magnitude and phase for the above filter at  $\omega = (0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{3})$

### 5 Question #5

- Design an FIR linear-phase digital filter that has the following approximate frequency response

$$H_d(\omega) = \begin{cases} 1 & \text{for } |\omega| \leq \frac{\pi}{3} \\ 0 & \text{for } \frac{\pi}{3} < |\omega| \leq \pi \end{cases}$$

- Determine the coefficients for a 6th order filter based upon a Hanning window.

- For the above filter, determine the gain value  $K$ , such that gain of the filter is unity (ie 1).

7

## 6 Question #6

A researcher in the Dept. of Biology has designed an experiment to investigate the effect of temperature on the number of ducklings hatched from a nest. Under each nest he has placed a temperature probe and he has decided to sample the temperature once per minute (assume no aliasing). Further more he has decided to average the present temperature reading with the past three readings to create a filtered temperature value,  $y(n) = \frac{1}{4}(x(n) + x(n-1) + x(n-2) + x(n-3))$ .

a) Given the implementation of his data acquisition and filtering, which periodic temperature fluctuations in his experiment will be eliminated and hence perhaps adversely affect his experimental results?

## 7 Question #7

Given the following transfer function

$$H(z) = \frac{0.2248 + 0.3299z^{-1} + 0.2248z^{-2}}{1.0 - 0.4601z^{-1} + 0.2388z^{-2}}$$

- Determine the poles and zeros and plot in the Z-plane.
- Sketch the magnitude response at  $\omega = \{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\}$
- Show a Direct Form II realization.
- Show a Direct Form I realization of this filter.



February, 1998

Time: 50 minutes

Textbook, Notes and Calculators Allowed

Tuesday, March 3, 1998

Time: 50 minutes

Textbook, Notes and Calculators Allowed

A casual filter is described by

$$H(z) = b_0 \left[ \frac{1 - 2b \cos(\frac{\pi}{4}) z^{-1} + b^2 z^{-2}}{1 - 2a \cos(\frac{\pi}{4}) z^{-1} + a^2 z^{-2}} \right]$$

$$b = 0.95 ; a = 0.99$$

- Sketch the pole-zero pattern for this filter in the z-plane. Be sure to show the unit circle.
- From the pole-zero plot, sketch the magnitude response of the filter.
- From the pole-zero plot, sketch the phase response of the filter.
- Determine  $b_0$  so that the maximum gain is approximately 1.
- Show the direct form I and direct form II realizations of this filter. Be sure to specify all coefficients.
- What type of filter is this and what is the approximate bandwidth?
- Determine a new set of coefficients for the direct form I realization that will approximately double the bandwidth while keeping the ratio of pass-band to stop-band gain nearly the same.

1. If the following systems are not already minimum phase systems, convert them to minimum phase systems without changing the magnitude response and give the impulse response of the new system.

$$\text{a) } h(n) = [1 \quad -4 \quad 3]$$

$$\text{b) } h(n) = [-1 \quad 4 \quad -4]$$

2. Determine the minimum-phase system whose magnitude squared response is:

$$|H(\omega)|^2 = 101 + 10e^{j\omega} + 10e^{-j\omega}$$

3. Design a single pole, single zero, high pass filter with cutoff frequency  $\frac{19\pi}{20}$ .

UNIVERSITY OF SASKATCHEWAN  
COLLEGE OF ENGINEERING  
EE 484 - Signal Processing  
Final Examination

April 1997

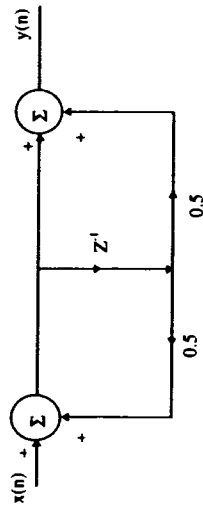
Instructor: J.E. Salt

Time: 3 Hours

Note: Textbook and notes allowed

Marks

1. Consider the filter below.



- (4) (a) Plot the pole zero pattern.

- (4) (b) What is  $y(n)$  if

i)  $x(n) = \cos \frac{\pi}{2} n$

ii)  $x(n) = \cos \pi n$

- (8) (c) Specify the resolution of the adders and multipliers (as well as the amount of truncation) needed to implement the filter in an application specific integrated circuit. The input is quantized and represented in 8 bit, two's complement format.

- (20) 2. Draw a flow graph of the filter implemented in the TMS320C31 assembler code shown below. Be sure to show the value and sign of all the coefficients. Also be sure to mark the inputs to a summer with a minus sign if you wish to subtract.

• include "initial.asm"

```

X_ADDR .SET 808048H
R_ADDR .SET 80804CH

.sect ".text"

MAIN: LDI3,BK
      LDI @BUFF_AD,ARO
      LDI @COEF_AD,ARI

      WAIT B WAIT

ISR: LDF 0,R0
     LDF 0,R2

     RPTS 2

     MPYF3 *ARO++,*ARI++,R0
     || ADDF3 R0,R2,R2
     ADDF3 R0,R2,R2
     LDI @R_ADDR,R0
     LSH 16,R0
     ASH -18,R0
     FLOAT R0,R0
     ADDF3 R0,R2,R2
     STF R2,*ARO++
     FIX R2,R2
     LSH 2,R2
     STI R2,@X_ADDR
     RETI

BUFF_AD .word 809900H
COEF_AD .word 809A00H

.start "flt_coef",809A00H
.sect "flt_coef"
.float 0.1
.float 0.2
.float 0.3
.float 0.4
.float 0.5
.float 0.6

```

```
.start "servect", 809FC5H
.sect "ser vect"
RET 1
B ISR
```

3. A filter was designed using the frequency sampling technique with the following matlab code. Two trials were done. A second frequency response statement was added after the program was run with the first frequency response statement. The matlab output for the two runs is shown after the code.

(20)

- Is the filter a linear phase filter and if so what type of linear phase filter is it.
- Plot the two impulse responses obtained from the two trials.
- Plot the two frequency responses you would expect from the two specifications.
- Plot the two phase responses as well.
- What the is bandwidth of the filter?

```
%parameters
N = 11; % filter length
w=[0:.1*pi/3*pi/3*pi/8*pi/pi];
A_w = [1 1 0 0 0]; %desired magnitude response at frequencies in w
A_w = [1 .95 .5 .1 0 0];
% calculation of the cosine matrix
n = [0:(N-1)/2];
cos_matrix = cos(w.*n*(N-1)/2);
% find the impulse response
two_H = inv(cos_matrix)*A_w';
H = two_H/2;
H((N-3)/2)=2*H((N-3)/2); % last element of two_H was not double so fix it now
H = H
```

# MATLAB COMMAND WINDOW

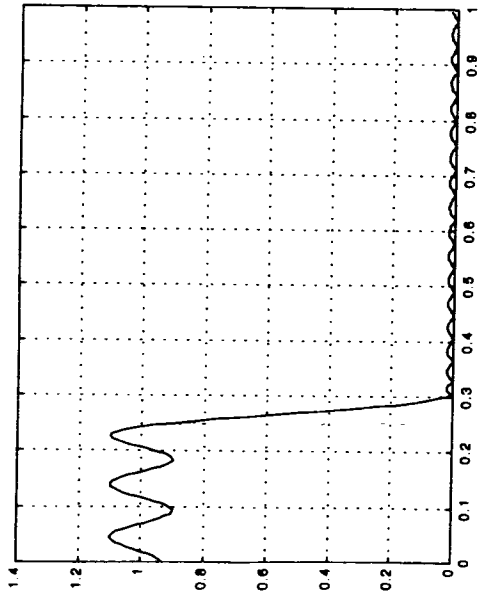
```
>> final_98_question
```

```
H =
-0.1101
-0.0068
0.2016
0.2500
0.1584
0.1318
```

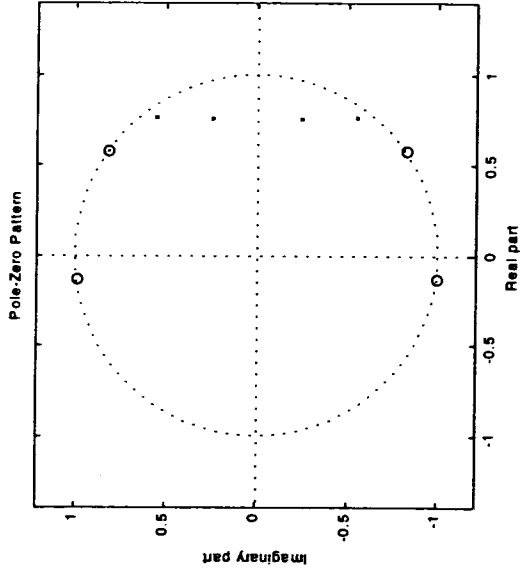
```
>> final_98_question
```

```
H =
-0.0100
-0.0075
0.0231
0.2000
0.2369
0.1575
```

- 4) a) What matlab commands were used to obtain the filter response shown below?
- (12) b) What is the approximate order of the filter?
- c) Are there any zeros located on the real axis. If so, state there approximate location? Be sure to explain your reasoning.



- 5) The pole-zero pattern for a low-pass filter is shown below.  
(12)
- What is the filter type?
  - What is the approximate stop band attenuation?
  - What is the approximate pass-band corner frequency?



6. (a) Find the DFT for the sequence

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

↑

(20)

- (b) Find the DFT of the  $N$  samples from  $n = 0$  to  $n = N-1$  of the sequence  $x(n) = a^{2n}$ .

- (c) Find the inverse DFT of

$$X(k) = [0 \quad -j \quad 0 \quad 0 \quad 0 \quad 0 \quad j]$$

- (d) Find the DFT of

$$x_3(n) = x_1(n) \odot x_2(n)$$

for  $x_1(n) = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$

$x_2(n) = [0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6]$

Time: 3 hours.

Instructor: Prof. J.E. Salt

Note: Text and notes allowed

April 1995

April 1995

**Marks**

- (10) 1. Find the discrete time Fourier Transform of

$$x(n) = \begin{cases} a^n & \text{for } n \text{ even; } n \geq 0 \\ b^n & \text{for } n \text{ odd; } n \geq 1 \end{cases}$$

- (10) 2. Find the discrete time Fourier Transform of  $Y(\omega)$  in terms of  $X(\omega)$  if  $y(n)$  is related to  $x(n)$  by

$$(a) \quad y(n) = \left( \sum_{k=-\infty}^{\infty} x(k)x(n-k) \right) \cos \omega_1 n$$

where  $\omega_1$  is a constant.

$$(b) \quad y(n) = x^*(n-1)e^{j\pi/2}$$

- (5) 3. (a) Find the steady state ~~frequency~~ response of the system with impulse response

$$h(n) = \left(\frac{1}{4}\right)^n u(n-3)$$

$$\text{if the input is } x(n) = \cos \frac{\pi}{3} n \quad Y(\omega) = \sum_{n=-\infty}^{\infty} y(n)e^{-j\omega n}$$

- (5) (b) The steady state output of a system when the input is  $x(n) = \cos \omega_0 n$  is

$$y_{ss}(n) = \left| \frac{1}{1 - 0.9e^{-j\omega_0}} \right| \cos(\omega_0 n + \theta(\omega_0)) \text{ for any } \omega_0$$

$$\text{where } \theta(\omega_0) = -3\omega_0 - \text{angle}(1 - 0.9e^{-j\omega_0})$$

What is the frequency response of the system?

.../2

**Marks**

- (5) 4. (a) Find the Z transforms of:

$$x(n) = \alpha^{2n} u(n) + \delta(n+10)$$

- (5) (b) Find the Z transform of  $y(n)$  in terms of the Z transform of  $x(n)$  if  $y(n)$  is related to  $x(n)$  by

$$y(n) = n x(-n)$$

The region of convergent of  $X(z)$  is  $r_1 < |z| < r_2$ .

- (5) 5. Prove that

$$\sum_{n=0}^{N-1} (\cos \omega_0 n + \sin \omega_0 n)^2 = N$$

for  $\omega_0 = \frac{\pi k}{N}$  for any integer  $k$ .

- (10) 6. Give the block diagram of a filter (showing all delays, sums and multiplier coefficients) that has a single pole at  $z = 0.5$  and a double zero at  $z = 1$ . The gain of the filter at  $\omega = \pi$  is 4.

- (5) 7. Find the inverse z transform of the stable system

$$X(z) = \frac{7z^2}{(z - \frac{1}{4})(z - 2)}$$

- (5) 8. (a) Is it possible to get a low pass filter with the 3dB down point at  $\omega = \frac{\pi}{4}$  and a

$$\text{relative gain } \left| \frac{H(\pi)}{H(0)} \right| = .2 \text{ with a single pole filter?}$$

If it is possible, give the location of the pole.

If it is not possible, either prove it or carefully explain it.

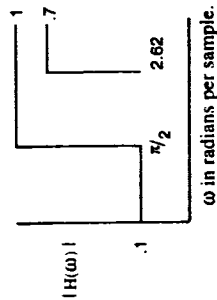
- (10) (b) Design a notch filter to remove the 60Hz component of a signal. The gain of the filter must be between .95 and 1 for all frequencies except those within 5 Hz of 60Hz. The sampling rate of the system is 2400 Hz.

.../3

April 1995

**Marks**

- (15) (c) Design a high-pass filter to the template given below.



- (6) 9. Classify the following system functions as linear or non linear phase filters? (A wrong answer will result in negative marks).

(a)  $H(z) = z^2 (z - z_1) (z - \frac{1}{z_1})$

(b)  $H(z) = \frac{z + a}{z - a}$

(c)  $H(z) = \frac{z^2 - a^2}{z(z + a)}$

- (2) 10. Is the system described by the system function below a minimum phase, mixed phase or maximum phase system.

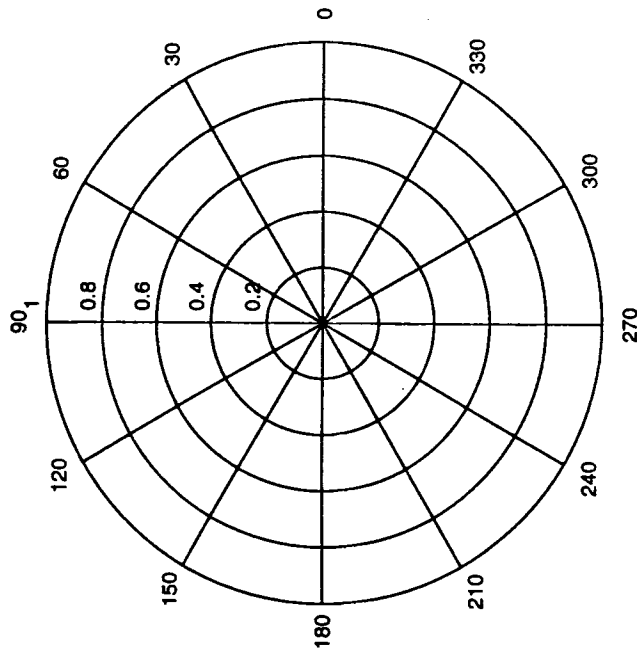
$$H(z) = \frac{(z - 7)(z + 3)}{(z - .5)(z + 2)}$$

$|H|$

- (2) 11. What is the 3dB bandwidth of the low-pass filter described by.

$$H(z) = \frac{(z - e^{j\pi/2})(z - e^{-j\pi/2})}{(z - .8)^2}$$

\*\*\* The End \*\*\*



Time: 3 hours.  
Instructor: Prof. J.E. Salt  
Note: Open Book

April 26, 1994

Marks

- (15) 1. Simplify the following expressions to the extent possible.

(a)  $\sum_{n=0}^{NM} \cos\left(\frac{2\pi n}{M}\right) \cos\left(\frac{2\pi n}{N} + \theta\right)$  where  $N, M$  are positive integers

(b)  $\sum_{n=0}^{\infty} (0.9 + j0.6)^n$

(c)  $\sum_{n=0}^{\infty} (3 + j3)^{-n}$

- (15) 2. Find the mathematical continuous time function or discrete time series, whatever the case may be, if their Fourier transforms are

(a)  $X(\omega) = e^{-j\omega} u(\omega)$

(b)  $X(\omega) = 1 + \cos\omega$

(c)  $X(\omega) = \begin{cases} e^{-j\omega} & ; |\omega| \leq \pi \\ 0 & ; \text{otherwise} \end{cases}$

Note: The argument  $\omega$  is used here in a general sense, i.e. it is also used for  $\Omega$  in which case it has units radians/sec.

- (15) 3. Find the Fourier Transforms or Fourier series coefficients, whatever the case may be.

(a)  $x(n) = \delta(n) + 7\delta(n-3) + \delta(n-6)$

(b)  $y(n) = \sum_{m=-\infty}^{\infty} x(n+9m)$ ; where  $x(n)$  is given in (a) above

(c)  $x(t) = \begin{cases} e^t & 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$

- (10) 4. (a) Is it possible for two filters with different pole/zero arrangements to have identical magnitude responses? Explain if it is or is not possible. If it is possible then give an example.

April 1994

Marks

- (b) Is it possible for two filters with different pole zero arrangements to have identical phase responses? Explain and give an example if such a filter is possible.

- (c) Is it possible to have filters that simultaneously satisfy a) and b)? Explain and give an example if such a filter is possible.

- (15) 5. (a) The system function of a filter is given by  $H(z) = 3 + z^{-1}$ . Find the output  $y(n)$  for input  $x(n)$ , where  $x(n)$  is given by

$x(n) = \cos\left(\frac{\pi n}{4} + 0.6\right) + 2$

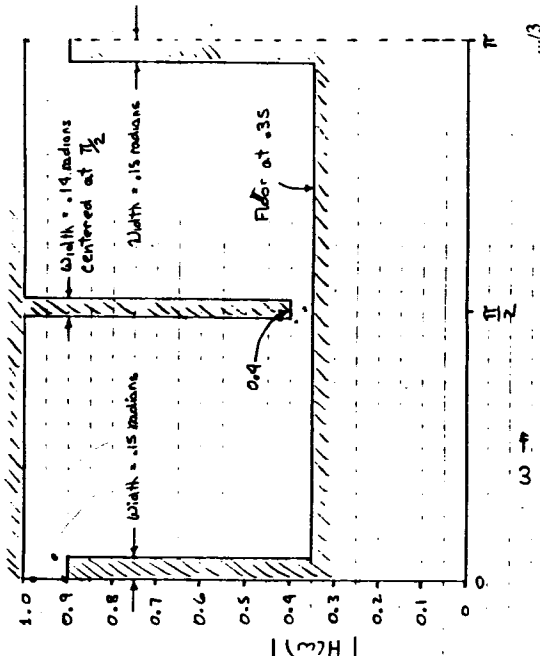
- (b) Consider the discrete time system with frequency response  $H(\omega) = 1 + e^{j7\omega}$ . Are the following three functions eigen functions of the system, and if so, what are the eigenvalues?

i)  $e^{j5n}$

ii)  $\cos\left(\frac{2\pi}{7}n\right)$

iii)  $\sin\left(\frac{3\pi}{28}n\right)$

- (15) 6. (a) Design a filter to the template below.

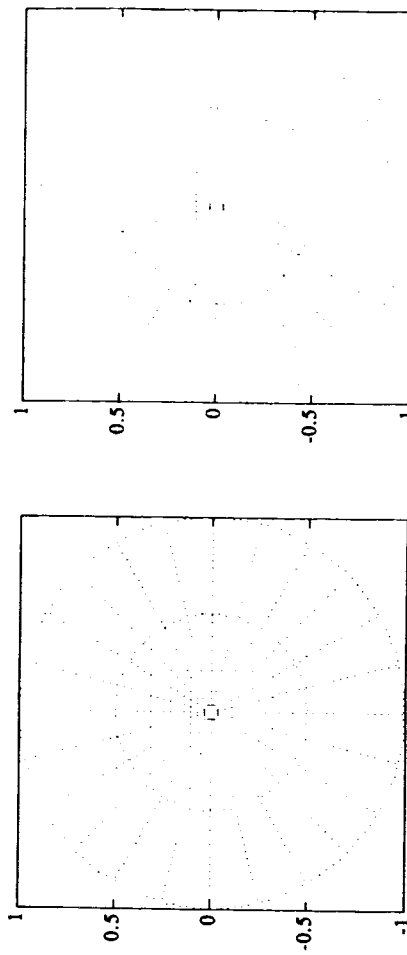
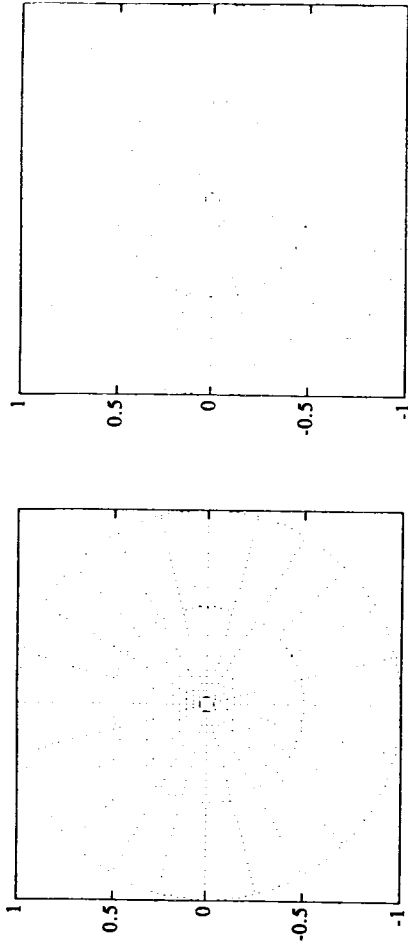


April 1994

Marks

- (15) (b) Design an implementable filter with a bandwidth of 2Hz and a notch at 60 Hz (i.e. the 60Hz response should be zero). The sampling rate is 6000 samples per second (i.e. after normalization the 60 Hz interference is at frequency  $\frac{60}{6000} = \frac{1}{100}$  Hz or  $\frac{2\pi}{100}$  radians). Be sure to clearly specify the location of the poles and zeros of your filter.

WORKSHEET



.... The End ....

(Worksheet attached)



EE 485: Communication/Transmission  
FINAL EXAMINATION, 9:00AM, April 29, 2002  
Time: 3 hours, closed book.

Examiner: Ha H. Nguyen

Permitted Materials: Calculator

Note: There are 5 questions. All questions are of equal value (with part marks indicated) but not necessarily of equal difficulty. Full marks shall only be given to solutions that are properly explained and justified.

1. (Ternary Modulation) Three equally probable messages  $m_1, m_2, m_3$  are to be transmitted over an AWGN channel with a two-sided PSD of  $N_0/2$ . The three signals used for transmission are:

$$s_1(t) = \begin{cases} 1, & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$s_2(t) = -s_3(t) = \begin{cases} 1, & 0 \leq t \leq T/2 \\ -1, & T/2 \leq t \leq T \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

- 2 marks (a) Sketch the three signals  $s_1(t)$ ,  $s_2(t)$  and  $s_3(t)$ .  
3 marks (b) What is the dimensionality of this signal set? Find one basis set for the signal space. Draw the signal constellation.  
2 marks (c) Draw the decision boundary and label the decision regions for the optimal receiver that minimizes the message error probability.  
1 mark (d) Which of the three signals is most susceptible to errors and why?  
2 marks (e) Compute the error probability given that the signal identified in (d) was transmitted.

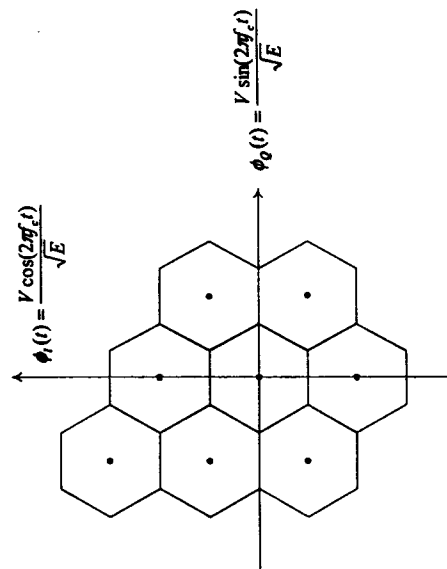
2. (AMI) Alternate-Mark-Invert is a binary line coding scheme. The output signal is determined from the source's bit stream as follows:

- If the bit to be transmitted is a 0, then the signal is 0 volts over the bit period of  $T_b$  seconds.
- If the bit to be transmitted is a 1, then the signal is either  $+V$  volts or  $-V$  volts over the bit period of  $T_b$  seconds. It is  $+V$  volts if previously a  $-V$  volts was used to represent bit 1,  $-V$  volts if previously a  $+V$  volts was used to represent bit 1. Hence the name and mnemonic for the modulation.

Now for the questions.

- 2 marks (a) Draw the three waveforms and a signal space representation of the above modulation.  
4 marks (b) Generally, the signal transmitted in any bit period depends on what happened previously. Thus there is memory and therefore a state diagram and a trellis. Draw a state diagram. As a hint, there are two states. Also a state is defined as what do you need to know from the past which together with present input (bit 1 or bit 0) enables you to determine the output ( $+V$ ,  $0$ ,  $-V$  volts). Label the transitions between the states with the input bit and the output signal.  
2 marks (c) Now draw the trellis corresponding to the above state diagram. Start at  $t = 0$  and assume that before  $t = 0$  the voltage level corresponding to a 1 is  $+V$  volts.  
2 marks (d) Assume that the source bits are equally likely and that  $V^2 T_b = 1$  joule. Using the signal space diagram of (a) and trellis of (c) sequence demodulate the following set of outputs from a matched filter for the first 3 bit intervals:  
 $r^{(1)} = 0.4$ ;  $r^{(2)} = -0.8$ ;  $r^{(3)} = 0.2$  (volts). (3)

3. (QAM) You are asked to design a modulation scheme for a communication system, and to conserve bandwidth it has been decided to use "QAM" modulation with an 8-point signal constellation. Unhappy with 8-ary PSK and 8-QAM because you feel that they do not use the available energy very efficiently, you decide to attempt a different signal constellation. Inspired by a tile design you notice in the local shopping mall, you propose the following signal constellation:



Assume each hexagon side is of length  $\Delta$ . Determine:

- 2 marks (a) The minimum distance between the signals (in units of  $\Delta$ ).
- 4 marks (b) The average transmitted energy per bit (in units of  $\Delta$ ).
- 3 marks (c) Draw a complete and neatly-labelled block diagram of the minimum error probability receiver. Show graphically the decision regions.
- 1 mark (d) Is it possible to do a Gray mapping (from a pattern of 3 bits to one symbol) for this constellation? Explain.

2. (CDMA) Consider a code-division multiple access (CDMA) system with two users. Every  $T_b$  seconds user 1 uses  $s_1(t)$  and  $-s_1(t)$  to transmit bit "1" and bit "0" respectively. Similarly, user 2 uses  $s_2(t)$  and  $-s_2(t)$  to transmit her bit "1" and bit "0". Both  $s_1(t)$  and  $s_2(t)$  are time-limited to  $[0, T_b]$  and have energy equal to  $E$ . The cross-correlation between  $s_1(t)$  and  $s_2(t)$  is given as usual by:

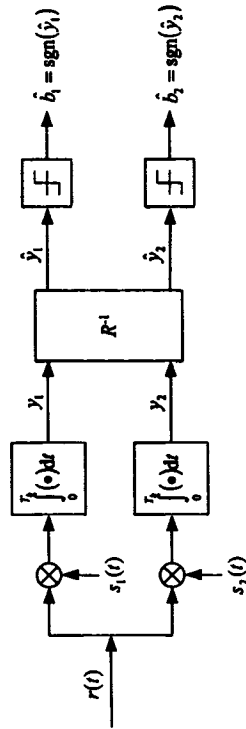
$$\rho = \frac{\int_0^{T_b} s_1(t)s_2(t)dt}{E}, \quad 0 \leq \rho < 1. \quad (4)$$

Since in a CDMA system, two users transmit over the same channel at the same time, the received signal in the first signalling interval is:

$$r(t) = b_1 s_1(t) + b_2 s_2(t) + w(t); \quad 0 \leq t \leq T_b \quad (5)$$

where  $b_1$  and  $b_2$  take on the values  $\{+1, -1\}$  with equal probabilities, and  $w(t)$  is AWGN with a two-sided PSD of  $N_0/2$  (watts/Hz).

- 4 marks (a) Consider the following block diagram of a receiver (known as the *decorrelating detector*) for the demodulation of  $b_1$  and  $b_2$ .



Obtain the expression for  $y_1$  and  $y_2$  in terms of  $E$ ,  $b_1$ ,  $b_2$ ,  $\rho$  and the noise components. Let  $n_1$  and  $n_2$  be the noise components in  $y_1$  and  $y_2$  respectively.

- 4 marks (b) Let  $R$  be the correlation matrix, defined as
- $$R = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \quad (6)$$

Then  $\hat{y}_1$  and  $\hat{y}_2$  can be computed based on the following relation:

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} = R^{-1} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad \text{where } R^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix} \quad (7)$$

Show that  $\hat{y}_1$  does not depend on  $b_2$ , the signal from user 2.

- 2 marks (c) Compute the mean and variance of the noise component in  $\hat{y}_1$ . *Hint:* Need to find the means and variances of  $n_1$  and  $n_2$  and the correlation between  $n_1$  and  $n_2$ .
- 10 marks 5. Do either (a) or (b). If you do both, the part with higher mark will be counted.
- (a) Describe and compare the following digital modulation schemes: BPSK, QPSK, OQPSK and MSK. (Concentrate on signal constellation, bandwidth efficiency, bit error performance).
- (b) Describe and compare M-QAM and M-FSK modulation techniques. What modulation schemes are suitable for band-limited and power-limited channels respectively? Explain.